Subject: **Methodology – Zero Net Migration Simulation Using Cohort Change Ratios**
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There are many ways to perform a zero-migration projection. This document presents a simple method that is based upon cohort demographic balancing equations and cohort change ratios. It can be applied to any set of already-published population projections that that includes births, deaths, and population by single-year-of-age and sex for each calendar year. Simply put, it sets all projected net migration rates to zero, and relies upon the assumption that the age-and-sex-specific birth and death rates remain unchanged. The three steps are:

1. Estimate net migration as residual from the births, deaths, and population data.
2. Estimate age-specific rates for all components of change.
3. Set all the net migration-rates equal to zero, and then project cohorts using cohort change ratios (also called the Hamilton-Perry Method).

Equations

Cohort Demographic Balancing Equation

For discrete ages *a=0, 1, …, 99, 100* (top-coded at 100), two sexes *s=1* (male) or *s=2* (female), the single-year cohort demographic balancing equation for years t and t+1 can be written as

$$P_{t+1,s,a+1} = P_{t,s,a} - D_{t,s,a} + M_{t,s,a};\ \ a = 0, \dots, 98\,; \qquad (1.a)$$

$$P_{t+1,s,100} = \left(P_{t,s,99} - D_{t,s,99} + M_{t,s,99}\right) + \left(P_{t,s,100} - D_{t,s,100} + M_{t,s,100}\right); \qquad (1.b)$$

$$P_{t+1,s,0} = B_{t,s} + G_{t,s}\,. \qquad (1.c)$$

$P_{t,s,a}$ is population stock at time *t*, sex *s*, and age *a* to *a+1*. The other values are flows that are for people aged *a* to *a+1* between time *t* and *t+1*, but we index them at *t* to reduce clutter. *D*, *B*, and *M* are deaths, births, and migrants. *G* is for newborn migrants who were born somewhere else after time *t* and have since in-migrated.

Calculating Net Migration as a Residual

When the annual births, age-specific deaths, and beginning and ending populations are known, we can solve for migration using

$$M_{t,s,a} = \left(P_{t+1,s,a+1} - P_{t,s,a}\right) + D_{t,s,a};\ \ a = 0, \ldots, 98\,; \qquad (2.a)$$

$$\left(M_{t,s,100} + M_{t,s,99}\right) = P_{t+1,s,100} - \left(P_{t+1,s,99} + P_{t,s,100}\right) + \left(D_{t,s,99} + D_{t,s,100}\right)\,; \qquad (2.b)$$

$$G_{t,s} = P_{t+1,s,0} - B_{t,s}\,. \qquad (2.c)$$

Note that Eq. 2.b cannot separate the migrants aged 99 from 100 and older in year *t*, but that is not required for the zero-migration simulation.

Calculating the Projected Age-specific Rates

Calculate a series of age-specific rates as follows

$$d_{t,s,a} = \frac{D_{t,s,a}}{P_{t,s,a}}\ ; \qquad (3.a)$$

$$m_{t,s,a} = \frac{M_{t,s,a}}{P_{t,s,a}}\ ; \qquad (3.b)$$

$$b_{t,s,a} = \frac{B_{t,s,a}}{P_{t,2,a}}\ ; \qquad (3.c)$$

$$g_{t,s} = \frac{G_{t,s}}{P_{t,s,0}}\ \ . \qquad (3.d)$$

In eqs.(3.a) and (3.b), age references the age of the person who might die or migrate, whereas in eq.(3.c) it references age of the mother.

Note that these calculated rates may differ from rates used to produce the original projections. This may be due to many factors; for example, the original projections may have different assumptions about the timing of events during the year, or the initial model may have followed a logic that required separate birth rates for new immigrants.

Projecting the Population

The following equations will reproduce the original projected results:

$$P_{t+1,s,a+1} = P_{t,s,a}\left(1 - \ d_{t,s,a} + m_{t,s,a}\right);\ \ a = 0, \ldots, 98\,; \qquad (4.a)$$

$$P_{t+1,s,100} = P_{t,s,99}\left(1 - d_{t,s,99} + m_{t,s,99}\right) + P_{t,s,100}\left(1 - d_{t,s,100} + m_{t,s,100}\right)\,; \qquad (4.b)$$

$$P_{t+1,s,0} = \left(\sum\nolimits_a b_{t,s,a} * P_{t,2,a}\right) + \left(g_{t,s} * P_{t,s,0}\right). \qquad (4.c)$$

For the zero-migration simulation, simply set all $m_{t,s,a}$ and $g_{t,s}$ equal to zero and apply Eqs. (4.a-4.c) iteratively over t. Note that dividing eq.(4.a) by $P_{t,s,a}$ produces a cohort change ratio for already-existing cohorts[1]. This zero-migration method might therefore be considered an application of the cohort change ratio method with the migration component removed.

[1] See, for example, Swanson, D. A., Schlottmann, A., & Schmidt, B. (2010). Forecasting the Population of Census Tracts by Age and Sex: An Example of the Hamilton–Perry Method in Action. *Population Research and Policy Review,* 29(1), 47–63. https://doi.org/10.1007/s11113-009-9144-7 .