Kem C. Gardner
POLICY INSTITUTE
THE UNIVERSITY OF UTAH

The Utah Demographic and Economic Model

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“No model, no understanding.”
– Nathan Keyfitz

Foreword

Models certainly do not replace thinking, but rather inform thinking. We build models to organize thought processes, understand relationships, shed light on cause and effect, and, ultimately, help people make INFORMED DECISIONS^{TM}.

In a high-growth state like Utah, long-term demographic and economic projection models provide vital information that helps with education, transportation, water, and other policies. They count on the development, upkeep, and operation of these models to help them make smart decisions and, ultimately, help our state prosper.

Over the years, Utah has benefitted from a variety of long-term projection models. The Utah Process Economic and Demographic Impact Simulation Model (UPED) served the state well for 26 years (1975-2001). This model was developed by the Bureau of Economic and Business Research (now the Kem C. Gardner Policy Institute), in collaboration with the State of Utah.

Later, the State of Utah turned to the REMI Corporation, a national modeling firm, and adopted the well-known REMI model to produce the state's long-term demographic and economic projections. REMI was used until 2015.

Today, it's our privilege at the Gardner Policy Institute to lead Utah's long-term demographic and economic modeling efforts. With the help of many partners, we designed, programmed, and now implement the Utah Demographic and Economic Model System (UDEM). We produced our first demographic and economic projections in 2017 and have now documented the model in this technical manual.

We are committed to continuous improvement of our projection process and model system. As time and resources allow, we will incorporate additional data sources, innovations in theory and modeling techniques, and more formal integration of economic migration dynamics as well as land, water, and other constraints. We will also customize our analytical approaches, so we are better able to address emerging and evolving policy issues.

We have designed and built our models so that they provide a useful and understandable framework for thinking about and planning for Utah's future. These capture essential demographic dynamics and economic drivers that shape how the future may unfold. Our hope is that these analytical tools provide our communities and decision makers with a better understanding of the plans, policies, and investments that will create the best possible future for Utah.

The actions we take today impact the future we are responsible for creating. It's our privilege to be a part of this important work. Our models will never replace great thinking, but they will go a long way towards helping people make INFORMED DECISIONS™.

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Acknowledgements

A broad network of organizations and individuals have supported and participated in the creation and implementation of the current Utah planning projection process. We are grateful for the expanding and enduring partnerships and collaborations as well as the support of our work program.

The Utah State Legislature and Governor's Office of Management and Budget have provided the institutional and financial support that have enabled the Kem C. Gardner Policy Institute to design, build, and implement the Utah Demographic and Economic Model System (UDEM).

The David Eccles School of Business, the University of Utah, and the broader community of supporters have created and sustained the institutional capacity of the Gardner Policy Institute so that we can produce research that contributes to a better understanding of Utah and its policy and planning context.

The Wasatch Front Regional Council, Mountainland Association of Governments, Utah Department of Workforce Services, Envision Utah, and Brigham Young University made significant contributions to the technical development and production of these 2017 baseline projections.

Foundational datasets were provided by the Utah System of Higher Education, Brigham Young University, Utah Vital Records and Statistics, The Church of Jesus Christ of Latter-day Saints, and the Utah Population Database.

Those who provided essential detailed local conditions data and review of preliminary results are too numerous to list here. We do want to highlight the contributions of the Bear River Association of Governments, Uintah Basin Association of Governments, Five County Association of Governments, Southeastern Utah Association of Local Governments, and Six County Association of Government, and Utah Division of Water Resources.

There have been many scholars and policy visionaries who established the intellectual and institutional foundations for the Utah Process Economic and Demographic Impact Simulation Model (UPED) going back to the middle of the 20th century. The most recent keeper of the flame was T. Ross Reeve whose body of work has informed and inspired the current UDEM development.

We are grateful to so many for believing in us and in the possibilities of this work. It is indeed an honor to be entrusted with the responsibility of continuing to build on the tradition of applied demographic work in and for Utah.

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Introduction

People require a view of the future in order to act with purpose. A coherent and comprehensible picture of a likely future is the foundation for consistent planning. Governments plan investments in infrastructure based on the future expected population of an area. Entrepreneurs invest in industries that are anticipated to hold the most promise for the success of their innovations. Medical providers plan facilities, programs, and research based on the future size and characteristics of the population. There is a widespread need for data about the future economy and population in order to make informed decisions.

Populations and economies are interrelated and complex, and the future is inherently uncertain. A well-specified population model can illuminate these interrelationships, and facilitate solid analysis and projections. The ideal model distills the most essential features of a complicated phenomenon in a way that is useful, reproducible, and understandable. These types of models are challenging to develop and never perfect, but also indispensable for well-designed policies.

1.1 Introducing The Utah Demographic and Economic Model

In 2015, the Utah Legislature, in partnership with the Utah Governor's Office of Management and Budget, tasked and appropriated funds to the Kem C. Gardner Policy Institute at the University of Utah for the development and implementation of a demographic and economic projection model system. The Gardner Policy Institute developed a population projection model for the state and implemented it to produce 50-year population and economic projections for Utah and its counties, which were released on July 1, 2017.

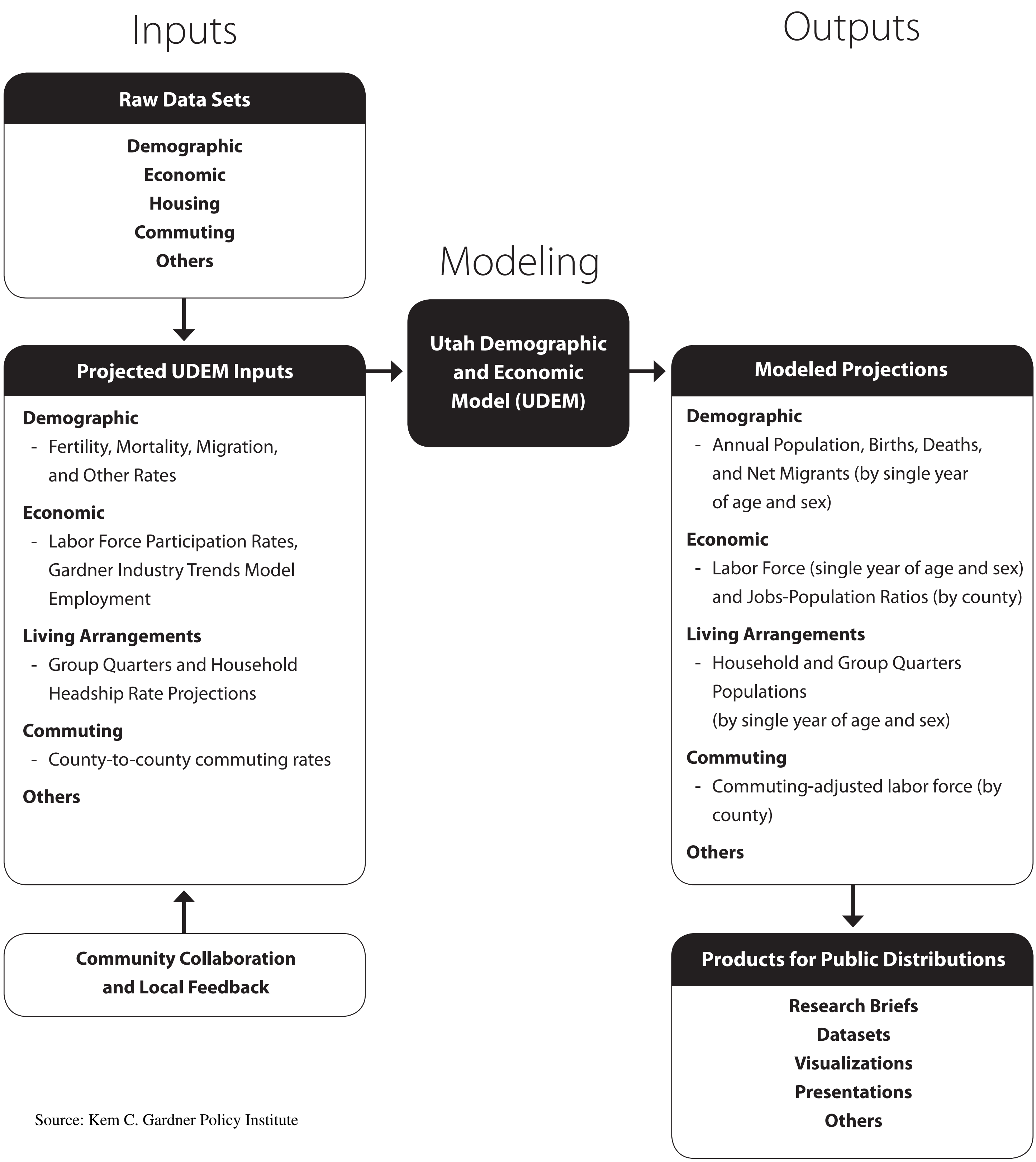
The resulting model is the Utah Demographic and Economic Model (UDEM), which estimates and projects the most essential demographic and economic characteristics of Utah and its counties into the future. The model builds upon past economic-demographic projection models designed for Utah, such as the State of Utah Demographic and Economic Projection Model System, and in particular the

Utah Process Economic and Demographic Model (UPED). UDEM is a current-generation model, whose outputs were designed to meet the present needs of policy planning and analysis efforts specific to Utah. Like previous models, it adheres closely to sound principles of demographic, economic, and biological science, but tries to eliminate unnecessary complexity to maximize returns on limited resources.

Figure 1.1 shows where the UDEM model fits within the context of the larger projection process. UDEM processes inputs and produces outputs. Those inputs—such as projected jobs, labor force participation, mortality, and fertility rates—are prepared beforehand from external data sources. UDEM is capable of incorporating alternative scenarios of the input data.

UDEM outputs are transformed into post-processed metrics which are reviewed and evaluated by the Gardner Policy Institute team as well as community collaborators. Based upon these reviews, the inputs are further refined to incorporate improved data, assumptions, and methods, and the models are rerun. This community collaboration has proven indispensable for improving the modeling inputs and processes by synergizing the collective expertise of Utah's research and planning community. Lastly, a set of projections is finalized, with the key outputs transformed and published as datasets, graphics, documents, and a suite of products.

Fig. 1.1 Utah Population Projection Process



Source: Kem C. Gardner Policy Institute

1.2 Explaining UDEM

UDEM is coded and operated in the R statistical software package by demographers and economists at the Gardner Policy Institute. But, the model is not a piece of software–it is a logical system of inputs and outputs. A clear understanding of the model logic enables policy-makers to more fully interpret the projections. The logic is encapsulated by algorithms that can be implemented in any software package, and should be accessible to anyone. Consistent with the Gardner Policy Institute's core values, shown in Figure 1.2, this manual presents that logic in two different ways.

Fig. 1.2 Kem C. Gardner Policy Institute Core Values

Committable Core Values

We subscribe to five committable core values that define who we are, how we conduct ourselves, and what comprises our work culture.

- **Responsibility to the community** – We exist to serve Utah and help our state prosper. While we make a contribution to regional, national and international public policy issues, we focus on assisting communities of interest within our state make informed decisions.
- **Research integrity and relevance** – We adhere to ethical principles and professional standards essential for responsible research. We value independent, relevant, meaningful, and understandable work.
- **Accountability** – We hold employees responsible for being continually productive in the work setting, being fiscally responsible, and delivering high quality products and services to the community, university, Eccles School, donors, and clients.
- **Collaboration** – We actively partner with people and organizations on campus and in the community to achieve our vision and mission. We believe teamwork among our staff will provide the best research and foster success.
- **Positive and passionate** – We employ people who are deeply committed to their craft and contribute to a positive and respectful work environment. We seek a congenial workplace where individual differences are both respected and celebrated, and we have fun along the way.

Source: Kem C. Gardner Policy Institute

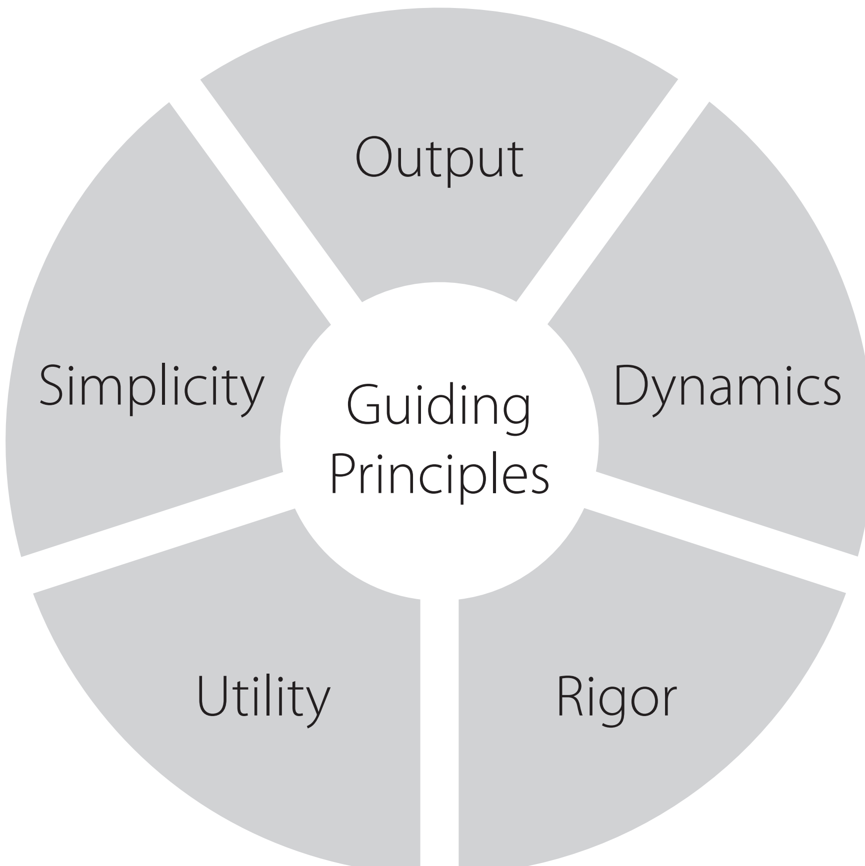
First, this manual communicates the basic logic of UDEM to an interested non-technical audience. This communication is critical for our accountability to the Utah community and will hopefully facilitate a more collaborative community engagement in the work. Our objective is to be transparent about how the projections are produced so that the results can be interpreted more easily and accurately.

Second, the logic of UDEM is presented in mathematical equations that would assist a technical expert in efforts to reproduce the model. This objective is critical for both scientific accountability and the model's long-term viability. Such documentation helps to satisfy expectations regarding research integrity and professional standards.

1.3 Guiding Principles

There are a variety of methods and models that can be utilized to project populations and economies. The design and specification of UDEM were determined by an integrated team of researchers and policy specialists with extensive experience in Utah policy planning. Technical and design decisions were made following a general set of guiding principles. These guidelines often overlap, but we can roughly divide them into five categories: (1) required outputs, (2) critical dynamics, (3) technical rigor, (4) general utility, and (5) calculated simplicity. These guidelines are discussed below, and summarized in Figure 1.3.

Fig. 1.3 UDEM Guiding Principles



Source: Kem C. Gardner Policy Institute

1.3.1 Required Outputs

The Utah State Legislature and Governor's Office of Management and Budget stipulated specific outputs for the long-term projections. The required demographic outputs include population by single year of age and sex, households, household population, group quarters population, and components of population change (births, deaths, and net migration). The required economic outputs are employment by major industry and labor force. Accountability is a core value of the Gardner Policy Institute; therefore, the modeling process should, at a minimum, produce these outputs for each of the 50 years in the projection period and for each of the 29 counties, and by the contract delivery date.

UDEM also produces outputs beyond the minimum requirements. The additional data points help identify anomalies during the quality control process and also set the stage for additional policy analysis beyond the minimum requirements. For example, UDEM produces detailed projections of the labor force by single-year of age and sex.

1.3.2 Critical Dynamics

Population size, characteristics, and dynamics are crucially relevant for planning processes. While some foundational population dynamics are shared by all populations, there are some that are unique to Utah. Research relevance is a core value of the Gardner Policy Institute. So, while the required outputs could be met in any number of ways, the design of UDEM must incorporate the unique demographic characteristics and dynamics of Utah if our research is to stay relevant. These are essential population dynamics in Utah:

- Cohorts. Demographers use the word "cohort" to represent a group of people born at a specific time. The term "generation" might be more familiar to most readers, who know of "Baby Boomers" and "Millennials." In a projection model, cohorts are maintained by making sure everyone who survives through the year becomes one year older.
- Migration. People move for many reasons, and those reasons are expressed differently according to the characteristics of persons, such as region, age, and sex. UDEM explicitly accounts for these types of migration:
  - *Labor-related migration* occurs when people move for employment opportunities. Young adults are the most likely to move for labor force reasons, and they often bring children in tow.
  - *Retirement migration* occurs in later life, and is especially concentrated in areas that provide amenities desirable to many older adults of independent means.
  - *LDS missionary migration* is a temporary migration, with young adults leaving for religious service, and returning within two years. In Utah, the number

of young missionary migrants substantially impacts population size, and also the number of births.
  - *Student migration* occurs when students move to another county to attend a college or university. These migrants generally exit the county four or five years later. If college students have children then the entire family moves after graduating. This type of migration can heavily impact the population size and composition of counties with college or university campuses.

- *Special populations* are distinguished from the general population because they have nonstandard population patterns. Often they are present because of an institution such as a university or college, correctional facility, or military installation.
- *Commuting patterns* account for the relationships between the place of work and place of residence. These are accounted for when commuters live and work in different counties. Factors that affect commuting patterns include cost and availability of housing and associated travel times.

1.3.3 Technical Rigor

Another Gardner Policy Institute core value is research integrity. We adhere to rigorous scientific and professional standards. In the UDEM work, we aim for empirical plausibility, logical consistency, and programming reproducibility.

- Biological, demographic, and economic plausibility.
  - Knowledge consistency. This means that the results should be consistent with historical trends and other known facts about how human populations and economies operate.
  - Mathematical consistency. This is a specific type of knowledge consistency, and means that known mathematical laws and relationships should be satisfied.
  - Face validity. This is sometimes called passing the "sniff test." The results should "make sense." However, here we have to be careful about outdated assumptions. Sometimes our life experiences lead us to believe one thing, but recent trends show something different. If conflicts arise, knowledge, and mathematical consistency have priority.
- Internal and external consistency.
  - Internal consistency. This means that one set of model results should not conflict with another. For example, the sum of the county numbers should equal the state number.
  - External consistency. UDEM's historical numbers should match other published historical numbers. For example, the total number of births in 2012 for Utah County should equal the number reported by the Utah Population Committee.

- Programming reproducibility. UDEM is coded in a software package. This ensures the model can be reused, easily improved upon, and potentially assessed by auditors. It can complicate user interface (at least initially), compared to out-of-the-box point-and-click software. The latter type of prepackaged software tends to hamper professional rigor by limiting the type of customization that is required to account for Utah's unique population characteristics and dynamics. Further, the "black box" character of commercial software limits our ability to interpret outputs.

1.3.4 General Utility

We designed and built UDEM to be capable of accommodating multiple purposes. If projections are presented as a static set of output, then people will not understand the conditions and factors that led to the results. This approach limits the utility of the model for policy planning, as well as our ability to explain results. We want a model that illuminates our understanding of causes and consequences.

UDEM is a structural model that consists of modules that are interconnected in logical ways. Changing inputs result in a different set of outputs that are understandable in terms of cause and effect. In contrast, some statistical models identify patterns in data and rely only upon historical trends. These inertia-based approaches focus upon the forecast number, not the process that produced it. Structural models, such as UDEM, have these advantages:

- Scenario analysis capability. In order to assess the impact of changing a policy, there must be some lever to pull that adjusts the model. This enables us to analyze different scenarios and to identify associated policies.
- Improved interpretation. The results of a structural model can be understood and explained by referring to inputs, model logic, and outputs. The relationships between key explanatory variables are made explicit and identified in terms of cause and effect. This provides a contextual foundation for understanding model dynamics and results.
- Collaboration. A structural model enables community partners to provide improved data, assumptions, and methods to further inform the projection process. This flexibility allows us to improve our analysis by incorporating additional data and factors that may not have otherwise informed our work.
- Streamlined upgrade process. UDEM can incorporate inputs that are produced in a variety of ways. For example, UDEM requires projected birth rates that can be developed by any number of methods. This enables us to improve upon the model by enhancing the birth rate module while the core model remains unchanged.

1.3.5 Calculated Simplicity

In an ideal world, we would be able to follow the above guidelines to the letter. But, in reality, no one has unlimited resources to create the perfect model. Deadlines limit our time, imperfect data limit the model, and budgets limit our resources. Because of these realities, we built UDEM to be as succinct as possible. Modelers call this principle "parsimony."

There are also scientific reasons to prefer parsimony. Analytic or mathematical solutions may not exist for complicated models. Also, complex models do not perform well in the presence of data errors. Each additional computation tends to propagate any errors in the input data.

Finally, we are better able to communicate and comprehend a parsimonious model, because there are fewer complicating contingencies to consider. This improved ability to communicate is critical for good policy planning and community engagement.

If two models perform equally well, we select the simpler one. "Simple" here does not mean less sophisticated, but indicates the model has fewer variables and calculations. Creating a model that balances the essential elements and dynamics of reality with conceptual and operational simplicity is a challenging enterprise. Our process of continuous model improvement is designed to further increase the model's adherence to the above guidelines while still maintaining parsimony.

1.4 Manual Layout

Although the design of UDEM is intentionally parsimonious, the technical mathematics are quite complex because model computations and equations are adapted according to available inputs. These more technical elements are included in this manual along with summaries that are expressed in more nontechnical language. This approach enables a broad audience to understand the basic mechanics of the UDEM model, while also allowing technical audiences to evaluate, replicate, or improve upon the work.

Part 1 is introductory and consists of three chapters. The present chapter contains a basic introduction to the UDEM model. Chapter 2 is an introduction to key demographic concepts used in the UDEM model. Chapter 3 overviews the logic of the model.

Part 2 involves the detailed model equations and operations, and is designed for technical practitioners. In practice, the Gardner Policy Institute maintains code that runs the model. But, if the model were to be built from scratch (as it originally was), the detailed logic is critical. Chapter 4 explains notation and introduces the software. Chapter 5 presents some commonly-used statistical adjustment procedures, which are utilized to maintain model consistency. Chapters 6 and 7 outline the death and birth processes. Chapter 8 documents the important cohort aging process. Chapter 9 explains the projection of migration as determined by the interplay of economic and demographic forces. Chapter 10 presents the household projection methodology.

The manual ends with suggested readings and references. Many of the ideas presented here have been informed by the prior work of other applied researchers and scholars. They have generally been published in so many places, that detailed attributions within the body of the text seem unnecessary. Therefore, to reduce clutter, references and suggested readings are placed at the end of the manual.

Chapter 2
Principal Concepts

UDEM mathematically processes demographic and economic inputs to generate outputs. Certain demographic and economic concepts can help readers better understand how UDEM produced those outputs, and also how to interpret them. This chapter introduces those concepts.

We define demographic concepts such as the projection timeline, populations, stocks, flows, demographic forces, group quarters, and households. We also introduce two important demographic laws: the Demographic Balancing Equation, and population aging. Economic concepts include the definitions of employment, jobs, labor force, and participation rates. Finally, we discuss how the supply and demand for labor can affect the relationships between employment and populations. Not everyone uses these same terms in the same way, and so we try to clarify and present the key definitions and relationships in a way that seems well suited for Utah.

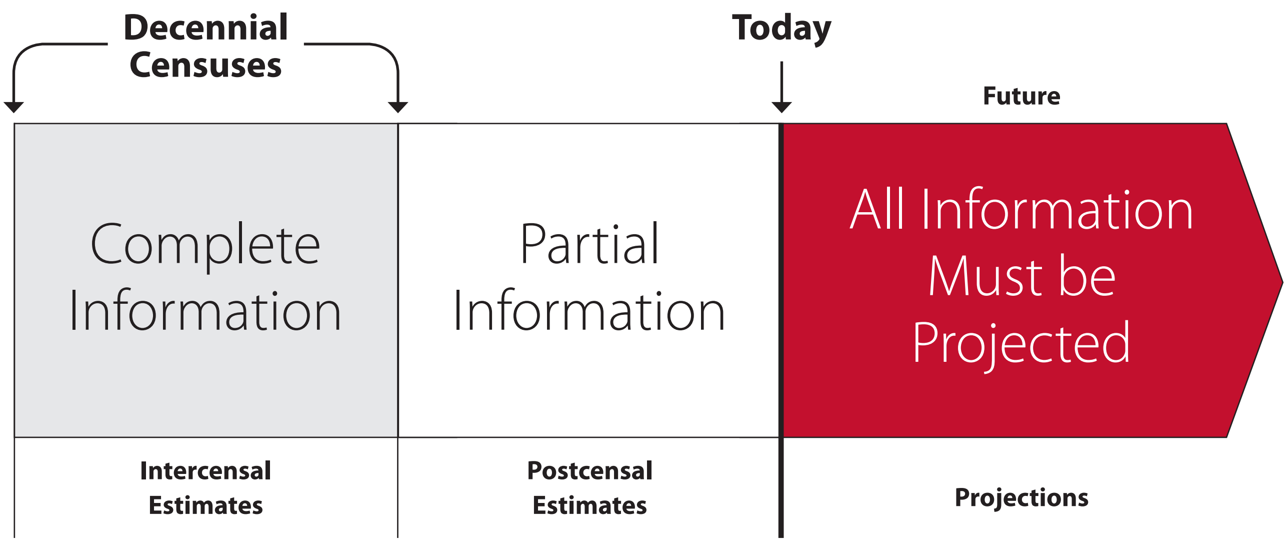
2.1 Demographic Concepts

Demography can be defined as the scientific study of populations and related processes. It also encompasses the practical application of the scientific concepts to different problems. The following concepts are particularly useful for understanding and applying the UDEM model.

2.1.1 Projection Timeline

When a projection model is run, it can be calibrated over a past time period when actual data are available. Since actual data will usually be more accurate than a projection model, we should use the data to the fullest extent possible. Therefore, in order to fully utilize the data, UDEM's logic changes depending on when the analysis is done. Figure 2.1 depicts a typical population analysis timeline.

Fig. 2.1 Population Projection Timeline



Source: Kem C. Gardner Policy Institute

The accuracy of any population analysis will hinge upon an accurate decennial census. This is because a decennial census, which aspires to provide a complete and accurate population count, provides the best population data available in the United States. Between any two decennial censuses, we can use special methods to compute intercensal estimates of the population at any time point. Subsequent to the most recent decennial census, there is usually some partial data available which can be utilized to estimate the complete data. That period with partial data is known as the postcensal estimates period. When projecting into the future, we have no data, so everything must be modeled.

A critical implication of this timeline is that intercensal estimates will usually be more accurate than postcensal estimates, which in turn should be more accurate than projections. Also, if the last decennial census was long ago, postcensal estimates and projections will probably be less accurate.

2.1.2 Population Definitions

We often think of a population as a group of people in a specified area at a specific point in time. As with many things, reality is more complicated than that. For example, typing "Utah population in 2015" into a search engine will probably yield a number of about three million people. But, at what time in 2015 was that number reached? And, who was counted? What about tourists who were visiting the area or missionaries and military personnel who were temporarily away? While these details might be unnecessary for a "back-of-the-envelope" calculation, they are of paramount importance when producing (and understanding) population projections.

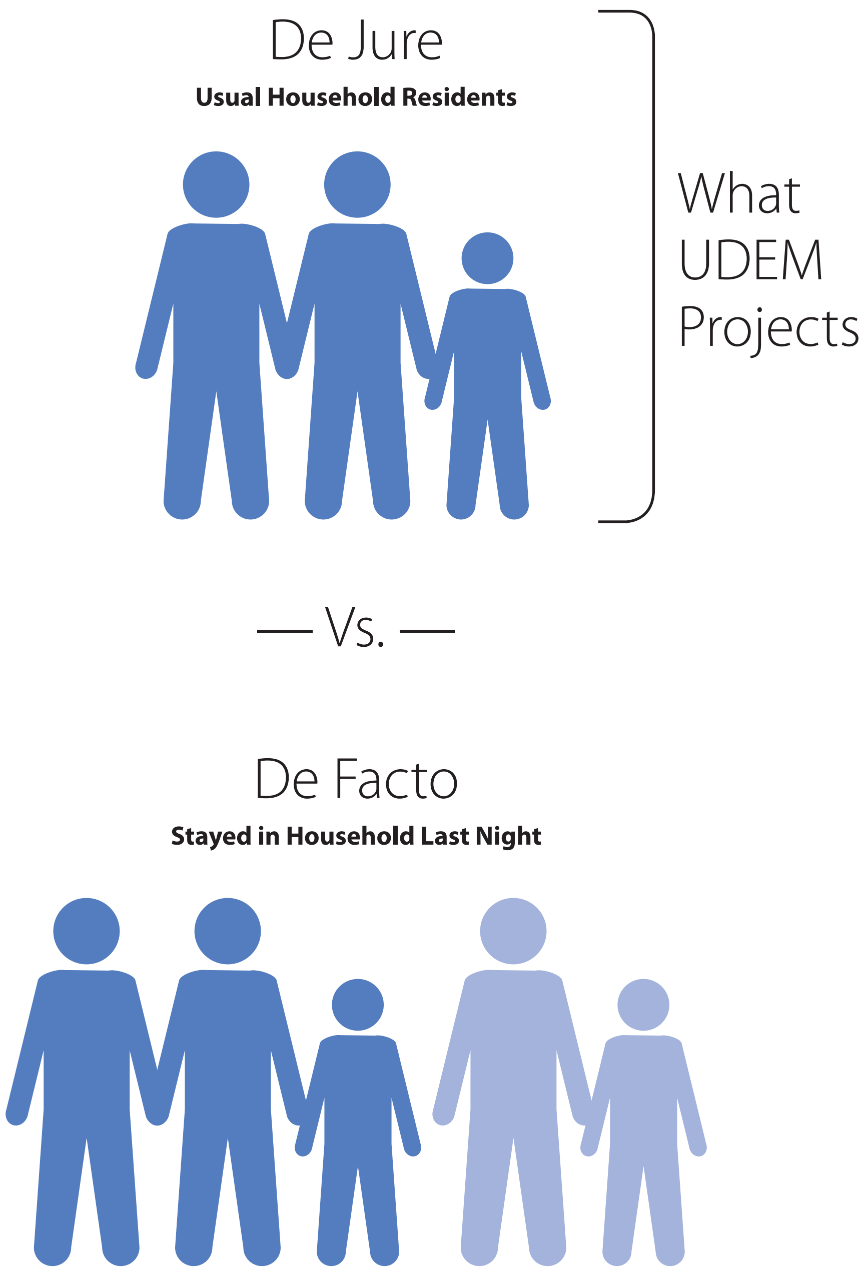
2.1.2.1 Resident Population

UDEM projects the "usual resident" population, similar to the Census Bureau. The Census Bureau defines usual residence as the place where a person lives and sleeps most of the time. For example, if an individual works in Salt Lake County but lives in their home in Utah County every night, Utah County is their usual residence. If the worker stays in an apartment Monday through Thursday in Salt Lake County, then returns to Utah County Friday through Sunday, Salt Lake County would be their usual residence. If an individual lives in a time share in St. George for 7 months of the year, but owns a home in Idaho and stays there for the remaining five months of the year, Utah would be their usual residence.

This delineation can ignore certain populations who frequent places and still use local resources, such as tourists and commuters, and can sometimes be at odds with an individual's subjective perception of "home." However, the definition is utilized for enumeration in the decennial Census, and is used in most demographic estimation and projection work.

The technical term for the usual resident population is the “de jure” population. This may be contrasted with the “de facto” population, which is the population actually present at a given point in time (including tourists and commuting workers). The de jure population is used for the apportionment of U.S. representatives, and often for allocating funds, and therefore is indispensable. The de facto population is essential for planning infrastructure, since utility and transportation capacity must accommodate tourists as well as residents. UDEM directly models de jure populations. Figure 2.2 summarizes these population definitions.

Fig. 2.2 Population Definitions



Source: Kem C. Gardner Policy Institute

In addition to the types of people, populations must be defined by time and geography. The relationships of populations with space and time are best understood in terms of demographic "stocks," "flows," and "forces."

2.1.2.2 Stocks

When an organization reports the population for a certain year, it is usually reporting a "stock." A stock is a count (or inventory) of something at a point in time within given geographical boundaries. The closest we can get to measuring stocks of the resident population is with the decennial census.

In reality, we cannot count everyone in an area at a single point in time, and so the decennial census considers that point of time to be April 1 of the decennial year. Often, data users want annual estimates of population for an entire given year. In order to have a reference time point over an entire year, July 1 (the year's "mid-point") is often used. This is not to be interpreted as the actual number of people actually present on that date, since certain seasonal patterns such as college enrollments or season-dependent labor can affect residences. Rather, it should be interpreted as an average over the year.

UDEM projects the resident population stocks for July 1 each year throughout the projection horizon. It also projects stocks of households, employment, labor force, group quarters populations, and other concepts. The stocks might require different levels of detail, or characteristics. For example, the population is disaggregated into single years of age and sex.

2.1.2.3 Flows

In contrast to stocks, a "flow" refers to change between two time points, and possibly two geographies. The main demographic flows are called the "components of change." They consist of births, deaths, and migrations. With births and deaths, a person begins to be counted, or ceases to be counted, during the time period. With migration, a person moves from one geography to another during the time period. A population stock can only change through these flows.

2.1.2.4 Forces

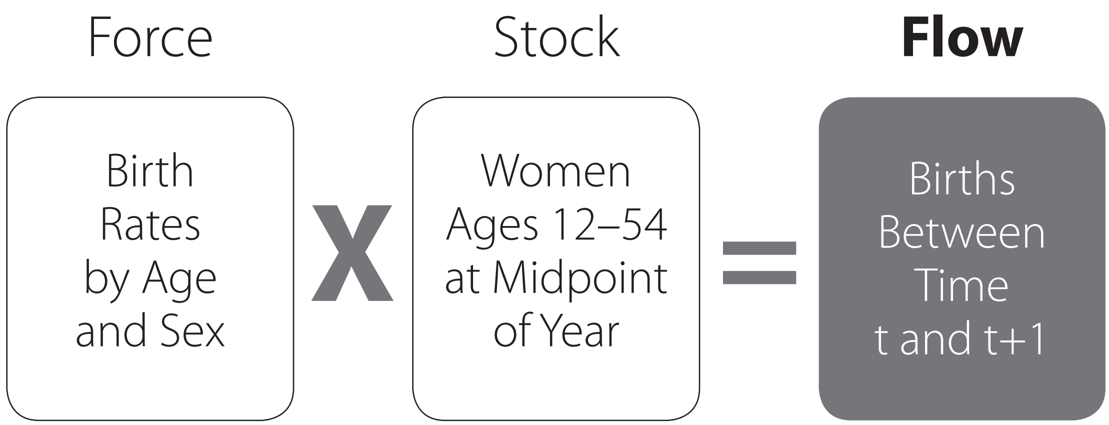
A "force" acts upon a population to affect the flows, and by extension the stock. These forces might be a combination of social, economic, biological, or other factors. A force is often measured by a "rate," and these rates are usually different for different ages. It is usually best to project the forces because they better measure the behavior of a population.

For example, because of the biology of aging, on average, younger people are less likely to die compared to older people. When we project the flow of deaths, we will want to consider the age composition of the stock, because on average, a younger population should have fewer deaths than an older one. Also, females on average live longer than males. A larger population should have more deaths than a smaller one. The force is the behavior of the population, usually expressed in the data as a rate, and can be applied to a future population of any size or age-sex composition. This maintains consistency between the flows of population and structure of stocks.

Some forces are more useful than others. Life expectancy can be considered a demographic force. It measures how long, on average, a person in a given stock will live. It is not realistic to project that everyone in the population will die at exactly that age. It is more realistic to project a different death rate for each age and sex group, and require those rates to be consistent with projected life expectancies. We call these age- and sex-specific death rates, because there is a different rate (force, or behavior) for each age and sex group.

Figure 2.3 illustrates these three relationships as applied to births. A force of age-specific birth rates is multiplied by a stock of women (of specific ages) to produce a flow of births between two time points.

Fig. 2.3 Stocks, Flows, and Forces Applied to Births



Source: Kem C. Gardner Policy Institute

2.1.2.5 Group Quarters

Living arrangements are also important for planning purposes. For example, a water planner might not only be interested in the number of people that need water, but also the number of housing units. Planning might differ for other types of structures such as college dormitories. The Census Bureau captures some of these differences by distinguishing between people who live in either "households" or "group quarters."

Group quarters are more communal environments such as shelters, correctional facilities, dormitories, or military barracks. Rooms in these group quarters often house several people. For planning purposes, it is useful to know the entire population living in group quarters. Anyone who does not live in group quarters is assumed to live in a household environment.

2.1.2.6 Households

The Census Bureau categorizes households as either family or nonfamily. They dwell in housing units such as apartments, detached single-family houses, or condominiums. Households often consist of more than one person, as is common with families. The average size of a household is called the "persons-per-household."

As mentioned above, households are particularly important for planners that need to provide services. The household is often considered the main "unit of consumption." For example, in many households an entire family shares utilities.

It is easy to confuse households with nuclear families. Many different groups, family and nonfamily, can comprise a household. Grandparents, older children, extended family, close friends, roommates, boarders, and a great diversity of other social relationships might form a household. These other social groups are extremely meaningful and important to understand, but simplifications are often required for large-scale planning. The household has been a useful designation, and is routinely used for policy planning. Anyone who does not live in a household environment is assumed to live in group quarters.

2.1.3 Demographic Laws

There are some processes that every population must follow. They are so important that we can consider them to be biological laws. Two such laws are the "Demographic Balancing Equation" and population aging.

2.1.3.1 Demographic Balancing Equation

Population stocks can only change through the flows of birth, death, and migration. Therefore any solid demographic analysis must follow the Demographic Balancing Equation.

This equation states that the stock at the end of a time period equals the stock at the beginning of the time period, plus the births and in-migrations, minus the deaths and out-migrations occurring over the intervening period. The equation is:

$$P2 = P1 + births + in\ migrants - deaths - out\ migrants \tag{2.1}$$

This equation can also be written in terms of "natural increase" and "net migration," and published results are often presented that way. Natural increase is the births minus the deaths, and net migration, is the in-migrants minus the out-migrants. Both natural increase and net migration can be positive or negative numbers, growing or shrinking the population.

$$P2 = P1 + natural\ increase + net\ migration \tag{2.2}$$

2.1.3.2 Population Aging

Every year a person must either die or become one year older. The age structure of the population is important for policy planning, and also a very important determinant of population change. As discussed above, the number of deaths will depend upon the age composition and size of a population. The number of births and migrations will also be affected by the population of women in childbearing ages and the size of the labor force. Therefore, aging should be included in a demographic projection model whenever possible.

The "Baby Boomers" provide an illustrative example. Birth rates were unusually high between 1946 and 1964. There were so many births that the generation has been called the Baby Boomers. Many of the Boomers have recently retired, or will retire soon, which impacts demand for various services and the size of the labor force. Failure to account for this "age wave" would paint a very unrealistic picture of the future population. In demographic terminology, such age waves are called "cohorts." UDEM strictly follows the natural laws regarding cohort aging. It also strictly follows the Demographic Balancing Equation requiring the components of change to be consistent with the population stocks. This type of model is called a "cohort-component" model. However, economic drivers also interact with a population, and UDEM incorporates this information as well.

2.2 Economic Concepts

The concepts of stocks, flows, and forces also apply to the economic concepts incorporated in UDEM. Jobs, employment, and labor force are stocks measured at a point in time. Total jobs is the demand for labor by employers. Employment is the number of jobs that are filled by employees. Labor force is the supply of labor and consists of both the employed and unemployed. The population has some labor force behavior, which is summarized by labor force participation rates. Additional economic forces included in UDEM are the unemployment rate and the rate of multiple job-holding or jobs per person (since a person can hold more than one job). When a person lives in a different county than they work, this is defined by commuting behavior. In UDEM, county-to-county commuting rates represent this force. The next section explores these economic concepts in greater detail.

2.2.1 Employment

Employment stocks, often referred to as "jobs," are closely related to Utah population growth. The current version of UDEM utilizes and produces employment stocks as measured by the US Bureau of Economic Analysis concept of employment. This measure of employment is a count of jobs, rather than a count of the people who hold them, and includes almost all formal sector wage and salary (payroll) employment, farm employment, military employment, and proprietorships and partnerships (self-employment). These job stocks pertain to the geography where the work is nominally performed, rather than the residence of those who perform the work. Both full- and part-time jobs are included, and no distinction is made between them. Informal sector work, paid through barter or cash and not formally reported, is not included.

2.2.2 Labor Force

The labor force consists of those in the population who are either employed or actively seeking employment. Those not seeking formally-measured employment (for example, the retired, full-time parents, those discouraged by long-time unemployment, and so forth) are not counted as part of the workforce. In contrast to employment, the count of labor force participants pertains to place of residence. Here, it is important to distinguish labor force from a demographic force, as described above. The labor force is best considered a stock - the number of people engaging in formally-measured labor-market behavior.

2.2.3 Labor Force Participation Rates

The labor force participation rate, in contrast, measures a force, or behavior. The rate is the fraction of those in the working age population who are in the labor force. UDEM utilizes labor force participation rates by single year of age and sex and defines "working age" as those aged 16 though 80. Such rates by group are simply the share of the population in each group that is in the labor force. The labor force participation rate is generally reported as a single number. Even if two populations have identical labor force participation rates by each age and sex group, the aggregate or reported rate will be much higher for a younger population than an older population. As the population ages, the aggregate rate declines.

2.2.4 Jobs per Employed Person

An employed person may hold one or more jobs. As noted above, these jobs may be full- or part-time. The average number of jobs per employed person is the ratio of the total number of jobs to the total number of employed persons; that is, the ratio of employment to the employed labor force.

2.2.5 Unemployment Rate

There are several metrics that measure the under-utilization of labor. One such measure, the unemployment rate (more precisely, the U-3 measure of unemployment, sometimes called the "headline" unemployment rate), is the fraction of the labor force that is unemployed. The unemployed are persons who are currently not employed but actively seeking employment.

2.3 Integrating Economic and Demographic Models

A demographic cohort-component model is based upon biological laws—there is no other way a population can change. If the inputs are correct, the model will always yield the correct result. However, the projected inputs are always uncertain, which introduces uncertainty into the model outputs.

Projections of the economy are even more uncertain. Not only are economic expansions and recessions difficult to forecast, but much economic activity is difficult to even define, measure, and model. Furthermore, future technologies will introduce unpredictable patterns of industrial composition and labor market behavior.

Combining demographic and economic models yields further uncertainty. They often affect each other in unexpected ways. Some demographers, particularly those working on a global or national scale, argue against using economic data for these reasons. Still, there is no denying that the state of the economy, especially as compared to other regions, significantly impacts the population. This is especially true for states and counties in the United States, where people are often willing and able to move across county or state lines for better employment opportunities.

Economic variables should be included in a population projection for Utah and its counties for three key reasons. First, the population is fairly small, which means even minor changes in employment opportunities can result in migration numbers that are large relative to the population, yielding major percentage changes in population size. Second, many policy actions aimed at affecting population will be related to the economy. In order to analyze the impact of an economic policy change on the population, the model requires economic variables. Third, population and economic data are often used together, and it can be helpful if they come from one integrated source. If economic and demographic projections are independent from each other, they will not be consistent. For example, a purely economic projection of labor force will not account for changing age structure that is evident in demographic projections.

One method for integrating population and economic models is by balancing the supply and demand for labor. People supply labor and demand employment. Employers supply jobs and demand labor. In UDEM, we refer to the level of employment necessary to satisfy the population's demands as "maintenance employment." It is determined by labor force participation rates, unemployment rates, rates of people working multiple jobs, the working age population, and other factors. If the supply of jobs is deficient, people may move elsewhere. But, if there is a greater supply of jobs than the population can fill, people will move in. In the rare event of a perfect match, no net employment-related migration would occur. While real life presents much more complexity, this model captures important elements linking population to employment, is easy to understand, and is particularly relevant to Utah and other Intermountain West states (where population and employment growth are closely linked). The model is therefore useful for Utah planning.

Figure 2.4 visually displays these critical concepts that link employment and population.

Fig. 2.4 Key Employment-population Concepts

Source: Kem C. Gardner Policy Institute

2.4 Models of Understanding

This chapter has presented demographic and economic concepts that researchers at the Gardner Policy Institute deem useful for understanding and applying the UDEM model. However, this is just one model of understanding—one which is useful for projecting Utah's population and economy.

A good projection model should be judged not only for its accuracy, but for its overall utility for a specific application and geography. Other models and definitions might be appropriate for other applications and geographies. Cohort-component projection models, like UDEM, require more data and time to implement than some less resource-intensive methods such as statistical extrapolation. Other definitions of employment might be appropriate for different projection models, or employment might be ignored altogether (particularly in multinational models where people cannot move as freely across borders). However, such models would not provide the interpretations or numerical outputs we deem necessary to make informed decisions for Utah and its counties.

2.5 Conclusion

While we might casually refer to the population and economy in everyday discussions, the precise definitions have significant implications for projection modeling. This also means that if policy-makers want their decisions to be well-informed, they must keep those definitions clear when interpreting the projected numbers.

Understanding key employment and population processes can further broaden our way of thinking. Illuminating such processes will allow policy-makers to identify additional policy levers they might pull to effect a change, and what changes might reasonably be expected. By themselves, numbers are meaningless. Only when we understand the specific context, the concepts that are being measured and modeled, and how those concepts operate, can we make truly informed decisions from demographic and economic projections.

Chapter 3
UDEM Overview

UDEM is a cohort-component population projection model with an economic driver. This chapter summarizes how the Gardner Policy Institute used UDEM to generate the 2017 baseline state and county population projections for Utah. The projection process included two different, but related models: a state model and a county model. The state model was run first. Deaths were calculated, then births, and then net migration. Next, the model was run for each county, constraining the age and sex totals for the county components of change to those of the state.

County population projections were then used to derive household projections, which were aggregated to yield state household numbers. Active collaborations with subject area experts, researchers, analysts, data providers, and community partners were ongoing throughout the projection process. While the state and county models do share a common analytic structure, there are key differences. The present chapter first details the state model. This is followed by a discussion of the county model that identifies essential differences from the state model. Finally, household projections are overviewed.

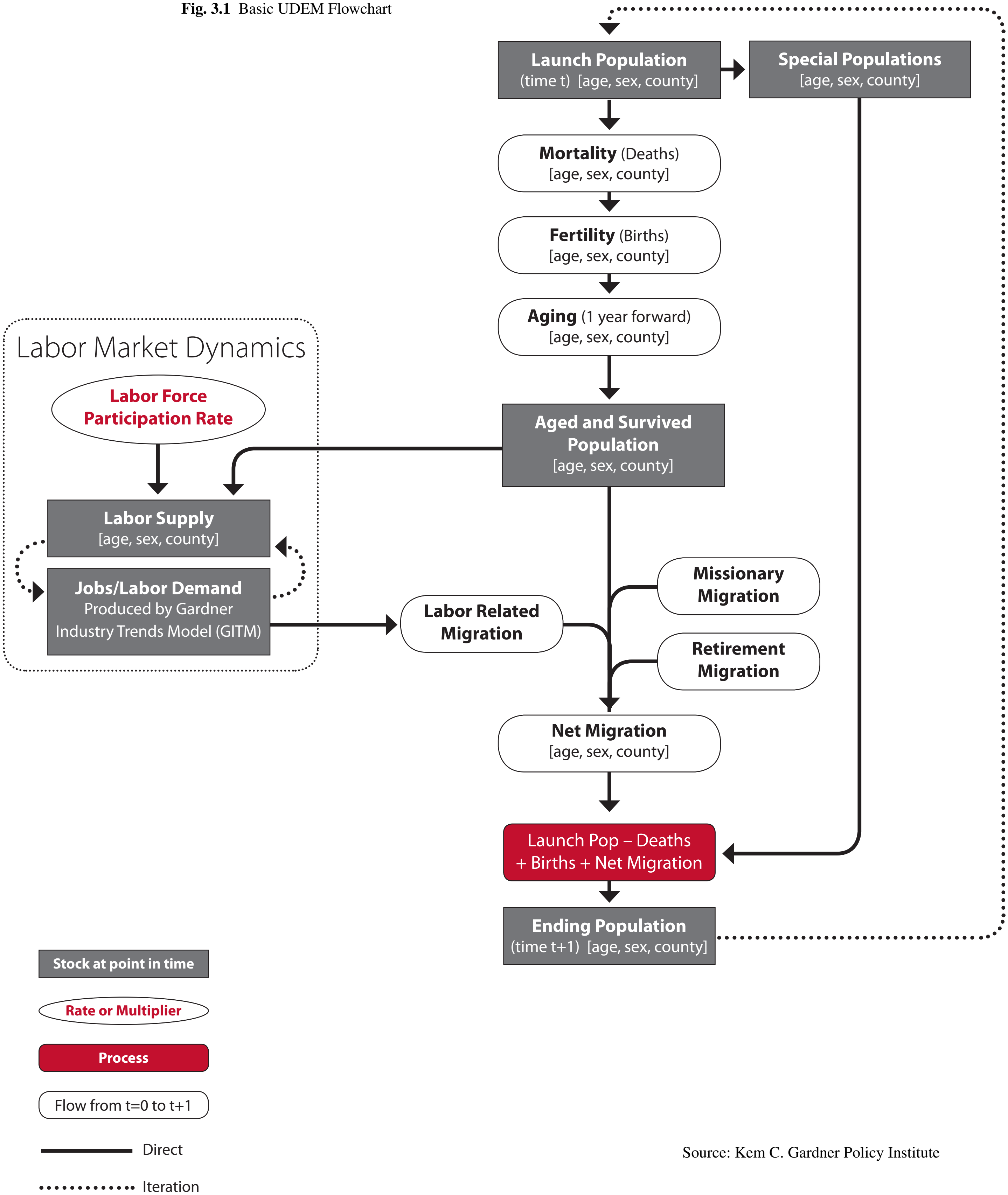
3.1 State Model

The initial or launch population in the state model was the resident population of Utah at the decennial census (April 1, 2010), for each single-year-of-age and sex. From this, UDEM first projected the population by single-year-of-age and sex three months to July 1, 2010. After the initial three month projection, an annual projection was produced for July 1 of each year through 2065. This was accomplished by projecting the detailed (single-year-of-age and sex) deaths, births, and net migrants during each one-year projection interval, consistent with the Demographic Balancing Equation,

$$P_{t+1} = P_t + B - D + M. \tag{3.1}$$

Here, t is an index for time. P_t is the population count at the beginning of the projection interval, and P_{t+1} is the ending population count. B, D, and M are the counts of cumulative births, deaths, and net migration occurring between times t and $t+1$. These are the demographic components of change, which were also disaggregated by age and sex. UDEM projected the components of change in a specific sequence: first deaths, second births, and third migration. The surviving population was also aged during each projection interval, consistent with a cohort-component projection model. Figure 3.1 provides a distilled overview of the model.

Fig. 3.1 Basic UDEM Flowchart

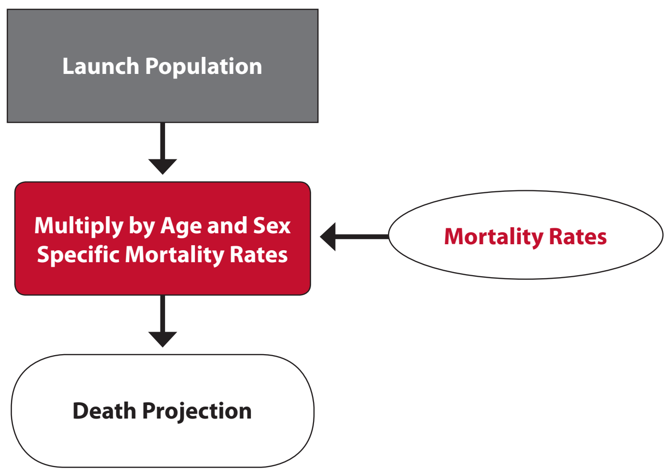


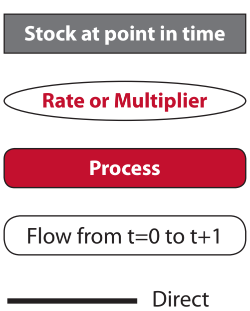
Source: Kem C. Gardner Policy Institute

3.1.1 Deaths and Births

Age- and sex-specific death rates were projected independently and used as inputs into UDEM. Those projected rates were applied to the starting population of each model iteration to obtain the number of deaths by age and sex. This is diagrammed in Figure 3.2.

Fig. 3.2 Deaths

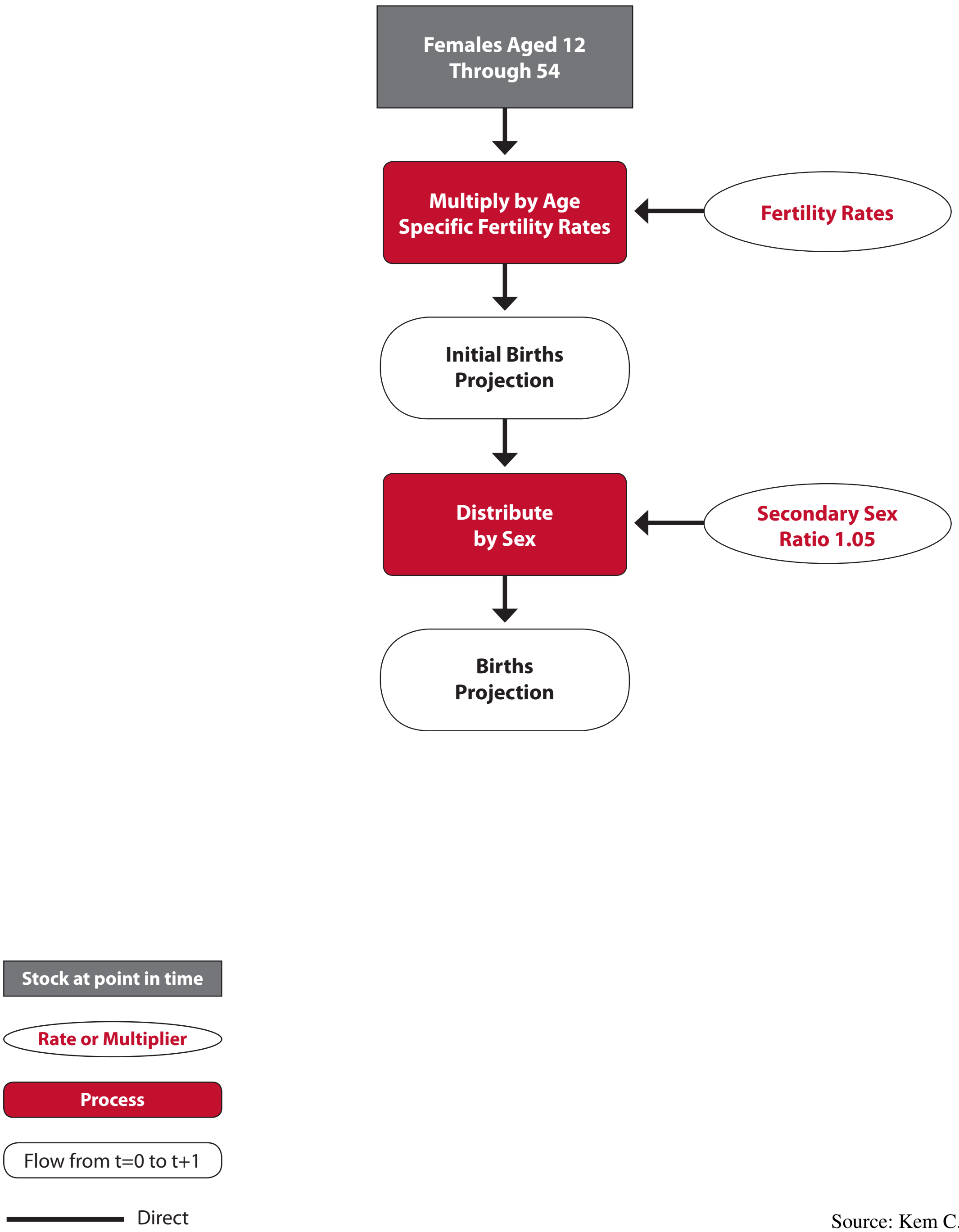




Source: Kem C. Gardner Policy Institute

Age-specific birth rates were also projected beforehand. These were applied to the population of survived women at the middle of the projection interval to determine the number of births. It was assumed that 48.8 percent of the newborns were female, and the rest male. Figure 3.3 displays the birth process.

Fig. 3.3 Births

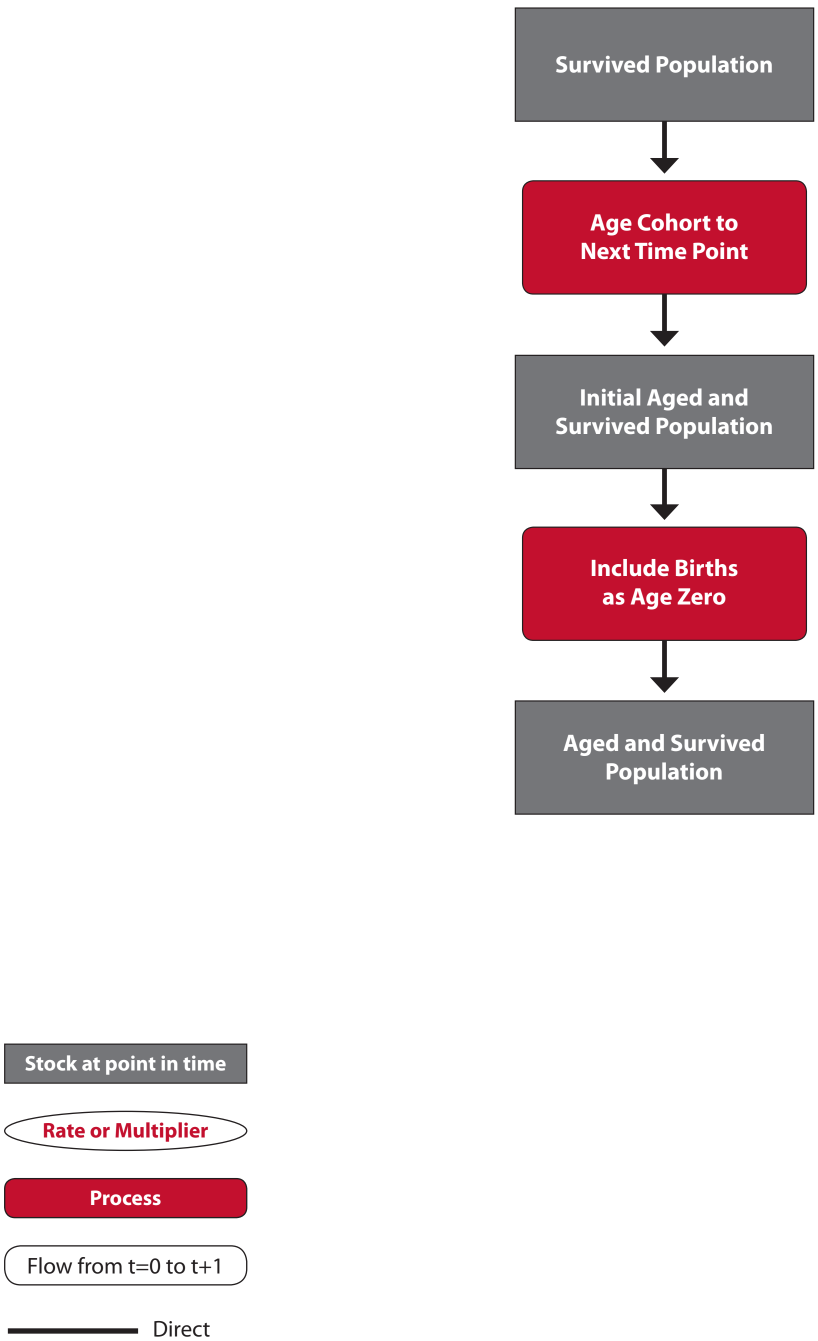


Source: Kem C. Gardner Policy Institute

3.1.2 Aging

After the births and deaths were projected, UDEM aged the population. The population that did not die during the interval was considered one year older at the end of the interval. New births were assigned an age of 0-1. Net migrants were assumed to be aged prior to the migration calculations. See Figure 3.4.

Fig. 3.4 Aging



Source: Kem C. Gardner Policy Institute

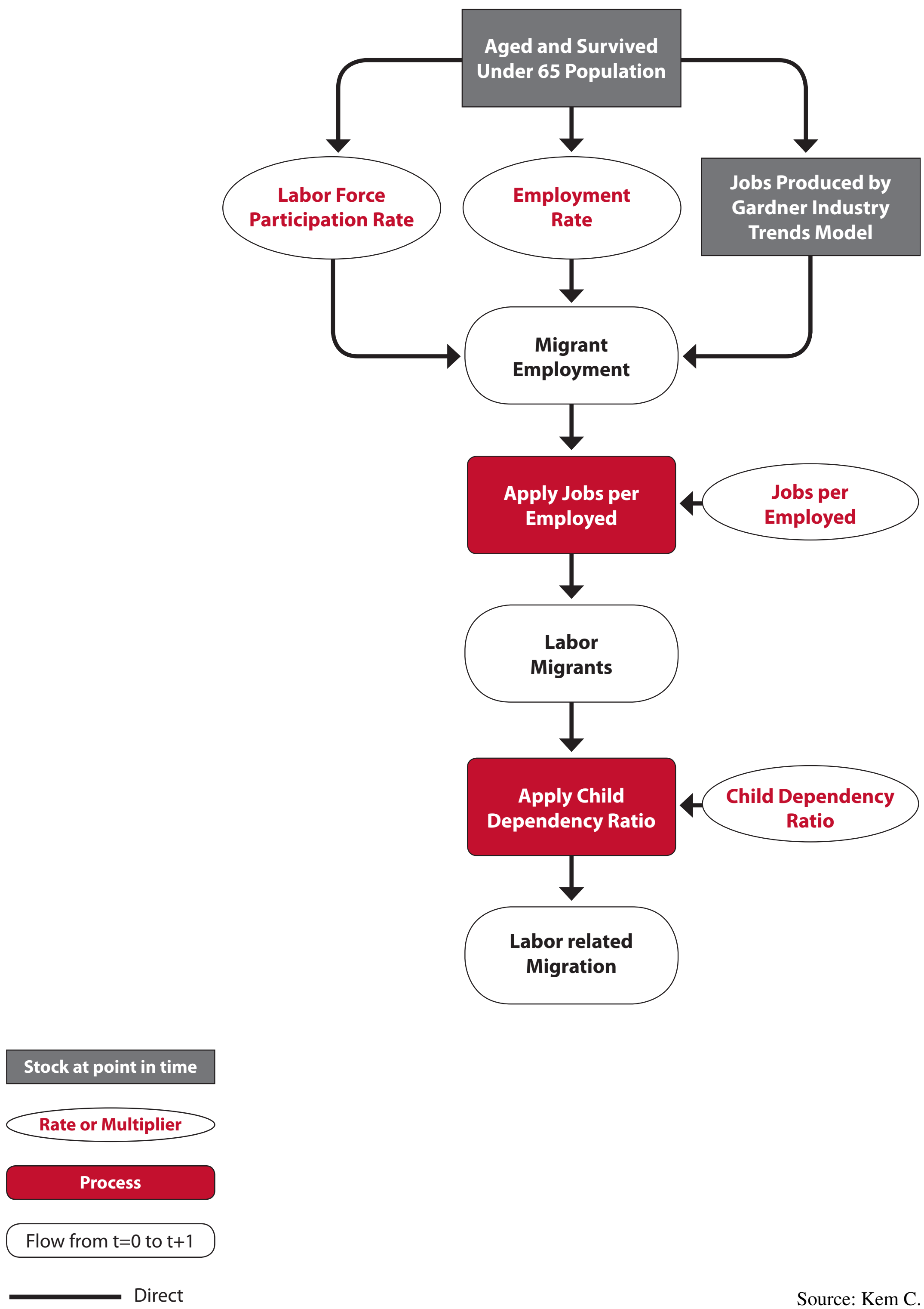
3.1.3 Migration

UDEM accounts for four migration categories: labor, retirement, missionary, and student. The first three were explicitly modeled. The fourth was modeled in combination with labor migration and special populations.

3.1.3.1 Labor Migration

Labor migration was determined in combination with baseline economic projections that were produced by the Gardner Industry Trends Model (GITM). Projected total employment, labor force participation rates, and unemployment rates were calibrated with historical data to estimate the maintenance employment--the number of jobs required to support the current population. UDEM then calculated the number of net labor migrants required to bring the supply of labor and demand for jobs into equilibrium. Next, the number of net labor migrants was multiplied by a childhood dependency ratio to account for additional child dependents assumed to migrate along with laborers. The labor migrants and their dependents were assumed to be half male and half female. Their ages were determined from historical net migration age patterns calculated between the 2000 and 2010 decennial censuses. See Figure 3.5.

Fig. 3.5 Labor-related Migration

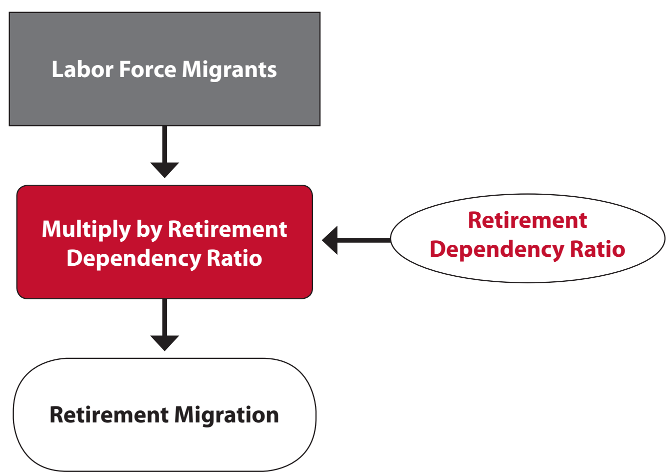


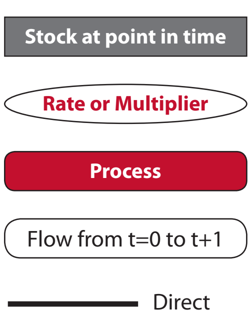
Source: Kem C. Gardner Policy Institute

3.1.3.2 Retirement and Missionary Migration

The number of labor migrants was multiplied by a retirement-age dependency ratio to determine a total number of net retirement migrants. This approach assumes that, at the state level, the link between the labor force and retirement-aged population will gradually shift in harmony with the aging population. The state-level retirement migration is summarized in Figure 3.6.

Fig. 3.6 State Retirement Migration





Source: Kem C. Gardner Policy Institute

Net missionary migration was determined from a combination of modeled past gross migration rates for missionaries and normative expected service times of two years for men and 18 months for women.

3.1.4 Special Populations

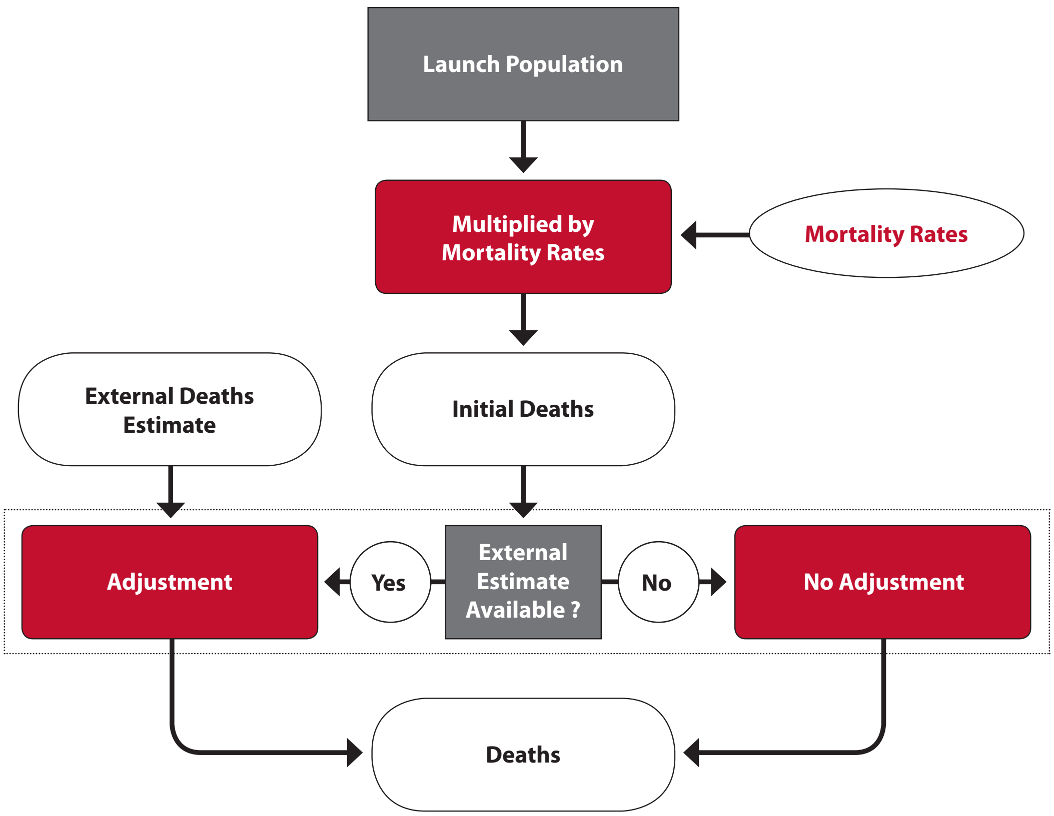
A special population is any population where the patterns of death, birth, and labor force participation observed in the general population do not apply. A population that would not be present unless a certain institution was present might also qualify. For example, a correctional facility population would not be present without the correctional facility. Special populations do not share the same birth and migration patterns as the general population. Rather than assuming a prison population will age in place for many years, it seems more reasonable to assume a regular flow of individuals will cycle in and out of the prison, keeping its age, sex, and population composition constant over time. Special populations can heavily influence projections, especially for substate regions, so we must account for them in our model. We identified special populations that included full-time students and many institutionalized (voluntary or involuntary) populations. Determinations were made on a case-by-case basis. Special populations were assumed to retain a constant size, age, and gender distribution over time.

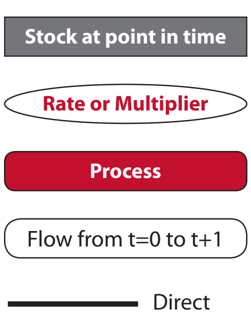
3.1.5 Controlling to Postcensal Estimates

The time horizon for projections can be subdivided into two periods. The first is between the decennial census and the year when the projections are conducted, and is often called the postcensal estimates period. The second is the future projections period. It is standard terminology to refer to present and past population values as estimates, and future values as projections.

During the postcensal period, it is common for external estimates of total population, births, deaths, and net migration to be produced. UDEM incorporated this additional information in order to improve the projections and ensure consistency with other results. In this set of projections, the births, deaths, and net migrants were adjusted to match totals published by the Utah Population Committee. An example diagram for deaths is presented in Figure 3.7. The patterns in these adjustments were then incorporated during the projection horizon to improve forecasts.

Fig. 3.7 Controlling to Postcensal Death Estimates





Source: Kem C. Gardner Policy Institute

3.2 County Model

The state projections were completed first, and then the projections were conducted for each of Utah's counties. This section details the key differences between the state and county models.

3.2.1 Commuting

Commuting rates measure the relationship between place of employment and place of residence. In UDEM, a commuting matrix is a data structure encapsulating the probability a worker that resides in any given county works a job that is recorded in any given county (possibly a different one). Conversely, a "reverse" commuting matrix shows the probability that a job located in any given county is filled by the resident of any given county. A commuting matrix distributes workers among all the counties, whereas a reverse commuting matrix redistributes jobs. Since Utah has 29 counties, each matrix for a given time point has 841 (29x29) data points. Figure 3.8 is provided as a reference to Utah's counties.

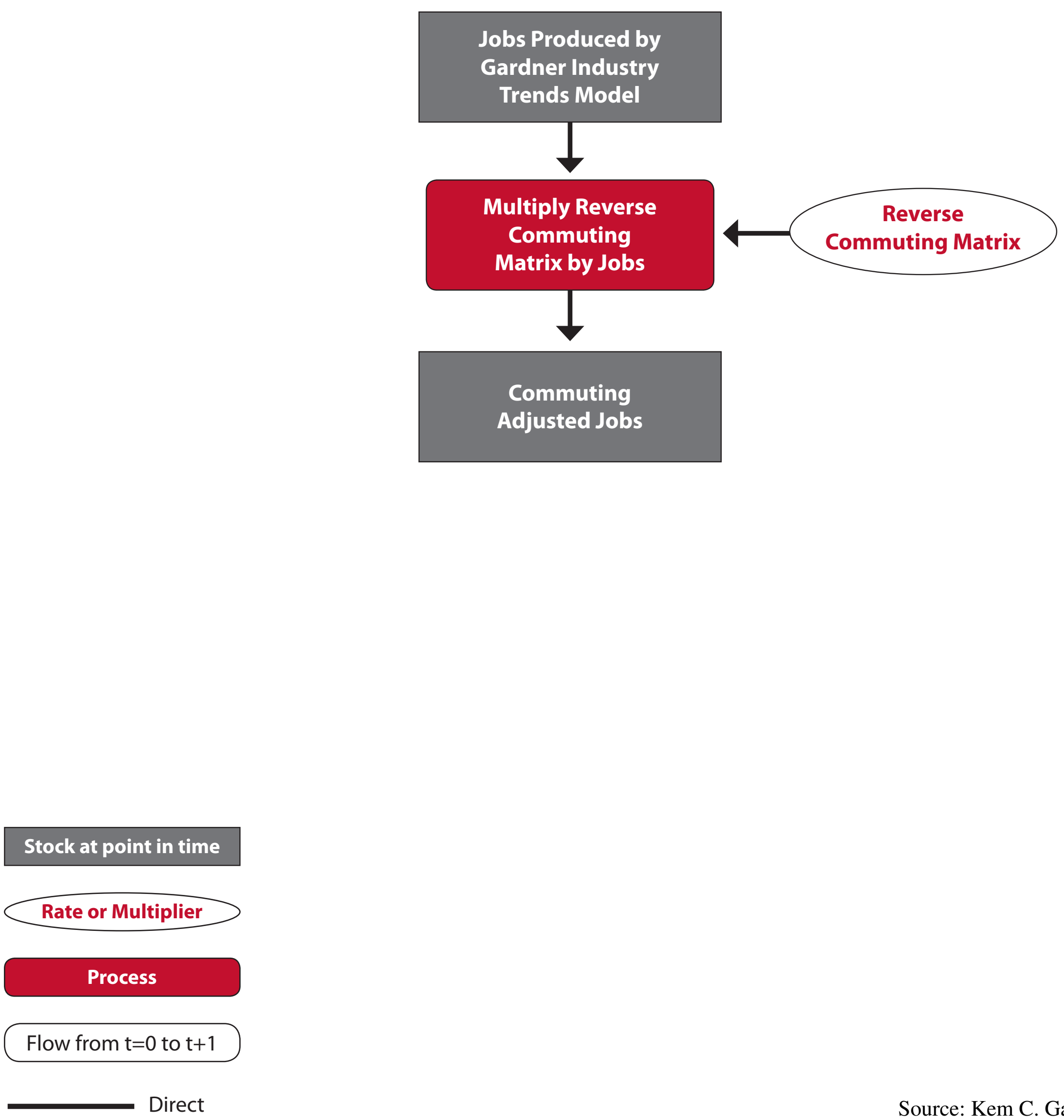
Fig. 3.8 Map of Utah Counties

Source: Kem C. Gardner Policy Institute

UDEM uses a reverse commuting matrix to redistribute the GITM-produced jobs to the workers' counties of residence, because the algorithm is simplified. At the county level, UDEM adjusted total employment between counties according to commuting patterns estimated and projected beforehand. This ensured that county population totals reflected individuals that live in one county but work in another. See Figure 3.9.

The reported county employment is the original, unadjusted GITM series; except in the four most populous counties along the Wasatch Front (Davis, Salt Lake, Utah, and Weber), where land use and transportation models, maintained by community partners including the Wasatch Front Regional Council, the Mountain Land Association of Governments, Envision Utah, and Professor Michael Clay at Brigham Young University, were used to cointegrate the regional projected GITM employment with commuting projections.

Fig. 3.9 Commuting Adjustments



Source: Kem C. Gardner Policy Institute

3.2.2 Retirement and Missionary Migration

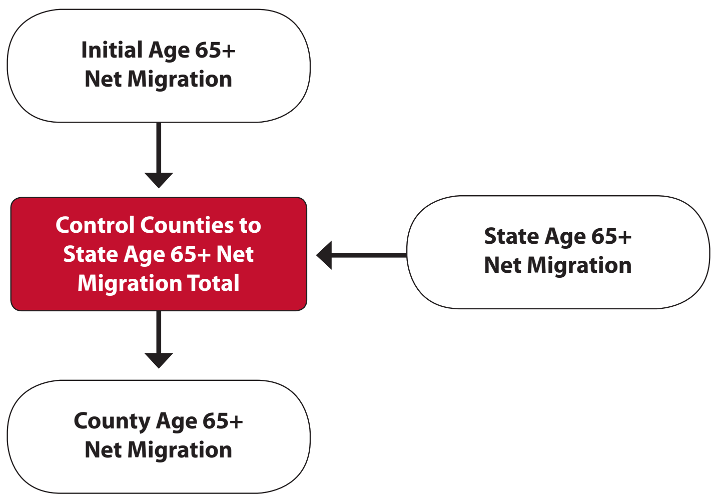
Patterns in retirement migration differ by county. State-level retirement migrants were allocated to counties based upon an analysis of the patterns observed between the 2000 and 2010 decennial censuses. Additional adjustments were made for certain counties where out-migration of retirees was expected to occur.

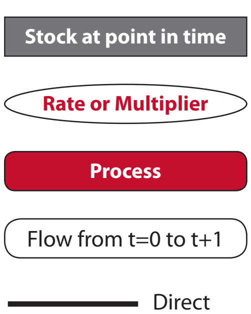
Also, in order to account for changing county retiree compositions, labor force participation rates during retirement ages were further adjusted within UDEM based upon labor-force patterns observed during the postcensal period. Missionary migration was not explicitly modeled at the county level, due primarily to data limitations. Instead, county-level missionary migration was determined iteratively by the model during the statistical adjustment to state totals.

3.2.3 Adjustment to State Totals

The county-level projections were made for each component (births, deaths, and net migrants) for a given projection interval, and these totals were then controlled to match the already-projected state components. This was accomplished through constrained econometric log-linear predictive models, estimated with an iterative proportionate fitting procedure. The predicted values maintained the counties relative demographic relationships as much as possible, but scaled their levels to be consistent with the state totals. See Figure 3.10.

Fig. 3.10 Controlling County-level Net Migration to State Totals



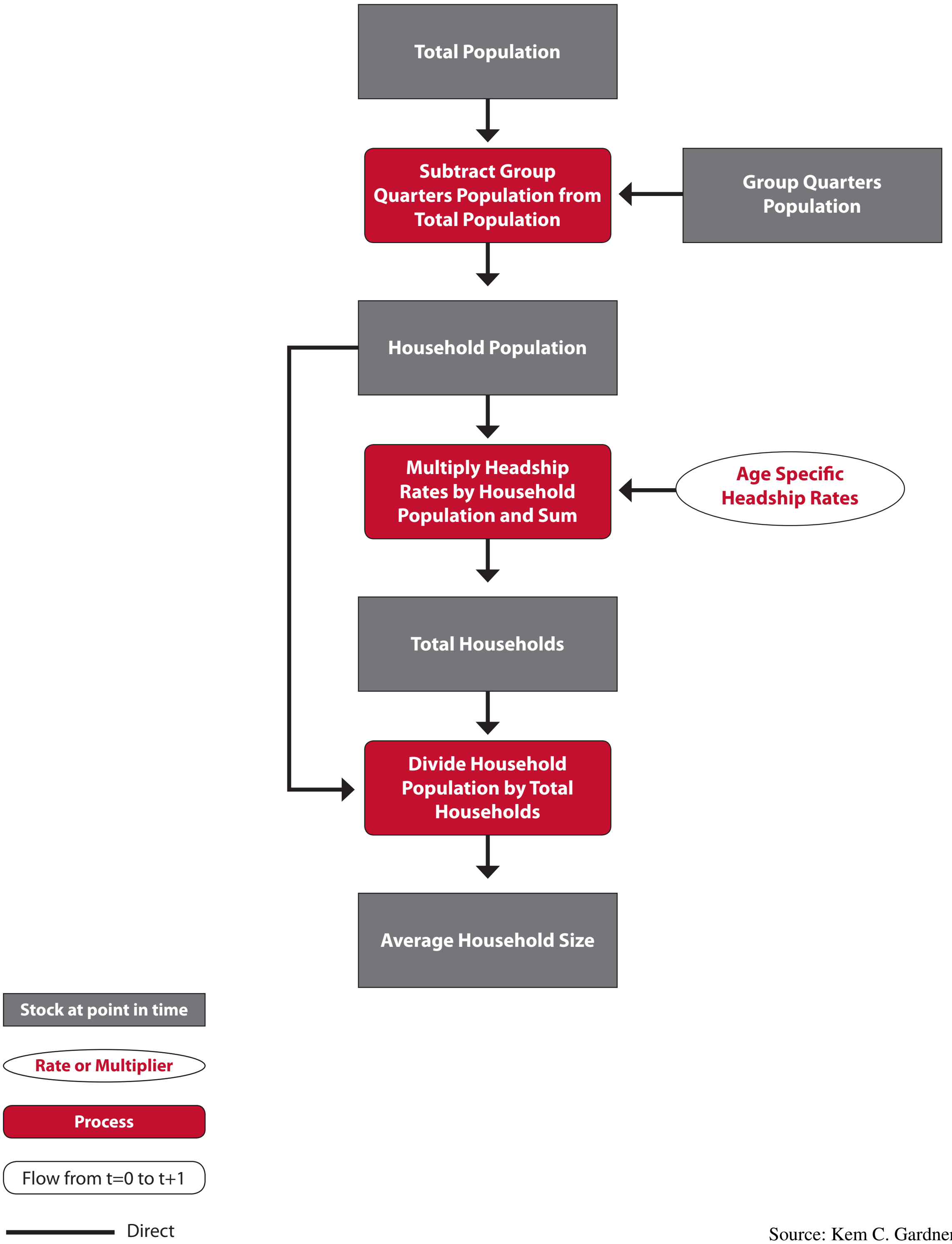


Source: Kem C. Gardner Policy Institute

3.3 Households and Household Size

Household projections were generated for each county after the population projections were completed. Annual growth in age- and sex-specific group quarters populations were projected. Then, household population was calculated within UDEM as the total population minus the group quarters population. Age-specific household headship rates from the 2010 decennial census were held constant each year and applied to the household population. This produced the number of households by the age of the household head. These were then aggregated to result in the total number of households. Dividing the household population by the number of households yielded the number of persons per household. The county values were aggregated to generate state totals. The household process is displayed in Figure 3.11.

Fig. 3.11 Households



Source: Kem C. Gardner Policy Institute

Part II
Technical Core of the UDEM Model

Chapter 4
Technical Introduction

This chapter provides important preliminary information for anyone wishing to implement the UDEM model. Documenting the technical aspects of UDEM requires a technical notation. This chapter explains the notation, and how it can summarize many calculations in one variable. It also introduces the software.

4.1 Notation

The unit of analysis is a region, age (single year), sex, and subpopulation group. In other words, this is the smallest unit that can be analyzed. Each model variable has a main name, with possible left and right sub and superscripts. Lowercase names are for rates, ratios, conversion factors, or some other force that acts upon a population. Uppercase names are for enumerated quantities such as population, jobs, or births. The enumerated quantities may be either stocks or flows.

In conventional demographic notation, left and right sub- and super-scripts are added to each variable name to indicate dimensions. We use this approach, although for preference, we custom-tailor our placement of the subscripts. A left lower subscript is the lower age range, and a left upper subscript plus the lower indicates the upper age range. Lower right subscripts indicate regions (e.g., i, or j). An upper right script indicates sex. Far lower right subscripts occasionally appear in parentheses, and indicate subgroups—this is important for differentiating general from special populations. The major benefit of the demographic style (compared to more common approaches of all right subscripts) is succinctness. The following example provides the general schematic:

$$ {}_{x}^{h}P0_{i(k)}^{s} $$

This variable indicates the starting population aged x to $x+h$ for county i, sex s, and subpopulation k, with a '+' being used to sum over a given dimension.

Thus, the total starting population is denoted ${}_{0}^{\infty}P0_{+(+)}^{+}$, instead of the more common $\sum_x \sum_i \sum_s \sum_k P0_{x,i,s,k}$

The dimensions and their coding are also summarized in Table 4.1.

Table 4.1 Notation Summary

Variable	Location	Definition
Sex=s	Upper right	1=Male, 2=Female
County=i	Lower right	Alphabetical: 1=Beaver, ... 29=Weber
Subpopulation=(k)	Far lower right	1=General Population, 2=Special Population
Bottom Age=x	Lower left	Numeric 0 through 100
Age Span=h	Upper left	Numeric 1 through ∞

A few more examples: ${}_{64}^{1}P_{1(1)}^{2}$ indicates the Beaver County "general" population (at the end of the projection interval) of females aged 64 to 65. ${}_{22}^{1}D_{+(2)}^{1}$ indicates the expected number of deaths to males aged 22 to 23 in the special populations for all of Utah. ${}_{27}^{1}lfpr_{29(+)}^{2}$ indicates the labor force participation rate for females aged 27 to 28 in Weber county. Table 4.1 summarizes the notation. When the upper left subscript is ∞, this denotes all the ages groups above the lower age group x. The UDEM currently topcodes age at 100 and above. UDEM is not currently a gross migration model. However, it occasionally is necessary to indicate flows between two counties (such as in commuting rates). In these cases, the county subscript i can be followed by an additional subscript j indicating a second county (possibly the same county), separated by a comma.

To eliminate unnecessary clutter, the '+' may be omitted when the context is clear. Also to omit clutter, indices of time are not included. Each projection interval is assumed to occur between times t and $t+1$. The starting stocks for the interval are at time t, then ending stocks at time $t+1$, and flows between times t and $t+1$. Forces are assumed to be indexed at time t, because they usually act upon some starting population to produce some flow—though this is not a hard and fast rule. When results are reported, flows are usually indexed to be at time $t+1$, because people tend to understand this better. Care has been taken throughout this manual to clarify reference time points when ambiguity arises. This is done by including $(t = T)$ after the variable name, where T is the time point. For example, $varname(t = 0)$ would be the variable at the decennial census.

Some demographic variables are specific to certain age groups. For example, retirement net migration only applies to ages 65 and above, whereas the missionary and labor-tied net migrations only apply to ages under 65. Total migration is the sum of these three. It is assumed that the unaffected ages are filled in with zeros prior to adding.

4.2 Software

Currently, the program is coded in R. The data are expected to be input from flat files. Some of the input data are read from a PostGres database. Some data is more sensitive and must be read from more restricted server disks. At the time the 2017 model projections were produced, there were three main R scripts. The first defined R objects to simplify the projection code, read the input data, initialized the necessary R objects from the input data, and included some smoothing of input data and synchronizing of rates from different sources. This first script also projected throughout the estimates period to obtain age and sex specific estimates. The second script ran the projections. The third script generated additional post-processing statistics, formatted output into a flat file, and exported to a database.

The scripts were run in that order, and the exact final code was archived in a repository. During the production process, changes were being implemented up through the last minute, so the code tended to become somewhat messy. The code is currently in the process of being streamlined for improved integration with this documentation, to facilitate a cleaner user interface.

4.3 Additional Notes

The equations presented here assume one year projection intervals. Between April 1 of the decennial census and July 1 of that year, a projection takes place with a three-month interval. Many of the equations have to be adapted accordingly, and this is done by multiplying by 0.25. It is not deemed prudent to add additional complexity to the equations in this document. One must simply be aware that whenever a change in populations occurs (i.e., deaths, births, migrants), the adjustment should be made.

Rounding is not done in the model, but the computer program is permitted to carry along values with as much precision as possible. Any rounding is performed when outputs are prepared for publication.

The iterative proportional fitting (IPF) procedure, discussed in the following chapter, is frequently used to bring consistency between county and state totals. But, this procedure does not work well in the presence of zero values. To remedy this situation, the model avoids zero population counts whenever possible by adding a minute constant of 10^{-6} to each zero value. The overall effect on the model outcomes should be trivial, particularly when compared to the gains it offers.

Chapter 5
Marginal Adjustment Procedures

Sometimes values need to be adjusted, so that they sum up to a new total. This is done through a statistical technique called a "marginal adjustment procedure." UDEM uses certain marginal adjustment procedures so frequently that they deserve their own chapter. These procedures are used to maintain age patterns in births, deaths, and net migration when matching published estimates. They are also used to ensure county-level values match state totals, while maintaining their demographic patterns. In short, they help provide external and internal consistency. The technical aspects are presented below.

5.1 Controlling to Totals

Sometimes values need to be adjusted, so that they sum up to a total (also called a margin). Some sources refer to this as as "controlling to totals," or a "marginal adjustment procedure." This chapter is not intended to discuss the statistical theory surrounding the technique—the literature is vast—but to present practical equations relevant to UDEM's implementation. Three types are used in UDEM: proportionate, uniform, and iterative proportionate fitting (IPF). The first two are unidimensional, in that they adjust to a single total. The latter is a generalization of proportionate fitting that works for totals along several dimensions.

5.2 Unidimensional Adjustment

In unidimensional fitting, it is assumed that there are q values represented by $n_1, n_2, \ldots, n_q$. In this manual, they are usually represented as demographic variables in several dimensions. They must be adjusted to one new total value W. This process is commonly called "raking." UDEM presently employs proportional and uniform raking.

5.2.1 Proportionate Adjustment

Function: $rake_p\left({}_x^hDV_{i(k)}^s, W, adjFactor\right)$

${}_x^hDV_{i(k)}^s$ is demographic variable in several dimensions. Take care to only include dimensions that should be summed over in the procedure. For example, if different adjustment factors are required for each county, do not include several counties in the input. Rather, call the procedure several times.

The values will be adjusted so that they sum to the new total W, with the difference being distributed proportionate to the observed distribution. The function will return either the adjustment factor or the adjustment factors, depending upon how the $adjFactor$ parameter is specified. The user should set the parameter to $TRUE$ if the adjustment factor is desired. Otherwise, the adjusted values are returned.

First, the adjustment factor af is calculated:

$$af = \frac{W}{{}_0^\infty DV_{+(+)}^+} \tag{5.1}$$

If $adjFactor = TRUE$, simply return af. Otherwise, the adjusted values, ${}_x^hAV_{i(k)}^s$, should be calculated as,

$${}_x^hAV_{i(k)}^s = af * {}_x^hDV_{i(k)}^s \tag{5.2}$$

and then returned.

5.2.2 Uniform Adjustment

Function: $rake_u\left({}_x^hDV_{i(k)}^s, W, adjFactor, allPositive\right)$

The values will be adjusted so that they sum to the new total W, with the difference being distributed uniformly among the categories. $adjFactor$ is a boolean parameter that should be set to $TRUE$ if the adjustment factor is desired. Otherwise, the adjusted values are returned. $allPositive$ is a boolean parameter that can require all resulting values be positive.

First, the adjustment factor af is calculated. This requires first determining the number of unique values contained in the demographic variable DV, defined as the number of bins, bin. Define the $length(.)$ of a vector as the number of its elements. Multiplying the lengths of all the dimensions yields bin.

$$bin = length(x) * length(i) * legnth(k) * length(s) \tag{5.3}$$

The additive adjustment factor is simply:

$$af = \frac{W}{bin} \tag{5.4}$$

If $adjFactor = TRUE$, simply return af. Otherwise, the adjusted values, ${}^{h}_{x}AV^{s}_{i(k)}$, should be calculated as,

$$ {}^{h}_{x}AV^{s}_{i(k)} = af + {}^{h}_{x}DV^{s}_{i(k)} \tag{5.5} $$

If $allPositive = FALSE$, return ${}^{h}_{x}AV^{s}_{i(k)}$. Uniform adjustment permits these AV values to have opposite signs. If $allPostive = TRUE$, any values that have become negative will be set to zero to generate a new variable ${}^{h}_{x}AV1^{s}_{i(k)}$. The difference in totals will then be removed from the positive values via the proportionate raking procedure.

First, create an indicator variable, pos for cases where AV has remained positive.

$$ {}^{h}_{x}pos^{s}_{i(k)} = \begin{cases} 1 & \text{if } {}^{h}_{x}AV^{s}_{i(k)} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.6} $$

The adjusted values $AV1$ can now be calculated as the original AV values, except replaced by zeros where they have become negative.

$$ {}^{h}_{x}AV1^{s}_{i(k)} = {}^{h}_{x}pos^{s}_{i(k)} * {}^{h}_{x}AV^{s}_{i(k)} \tag{5.7} $$

The total of $AV1$ will be too large, because the negative values have been replaced with zero. Controlling $AV1$ to the original AV totals using the proportionate adjustment procedure will solve the problem.

$$ {}^{h}_{x}AV2^{s}_{i(k)} = rake_{p}\left({}^{h}_{x}AV1^{s}_{i(k)}, {}^{\infty}_{0}AV^{+}_{+(+)}, FALSE\right) \tag{5.8} $$

Return $AV2$. Note that if none of the adjusted values AV were negative, the end result is the same as if $allPositive$ was set to $FALSE$. Note that, alternatively, one could perform another uniform adjustment (instead of proportionate); however, this would potentially yield more negative values, and the algorithm would need to iterate until all adjusted values are positive. However, we did not find a need to implement additional uniform adjustments.

5.3 Multidimensional Adjustment

Multi-dimensional values can be fit through iterative proportional fitting. The algorithm is too cumbersome to present here. There are many resources that describe the algorithms, and software packages that implement them.

However, we do have a functional notation for this documentation.

Function: $IPF\left(seed = {}^{h}_{x}DV^{s}_{i(k)} \mid \text{a set of marginal totals = another set of marginal totals}, \ldots\right)$

The seed is the set of starting values. IPF maintains all the pairwise odds ratios as well as possible, while matching targets. The pipe symbol "|" is commonly used in probability notation to denote "conditional on." The notation then suggests that DV will be fit as well as possible, given that one total is equal to another. The ellipses suggests that other totals can also be matched. An example is: $IPF\left(seed = {}_{x}^{h}DV_{i(k)}^{s} | {}_{x}^{h}DV_{+(+)}^{s} = {}_{x}^{h}DV9_{+(+)}^{s}, {}_{65}^{\infty}DV_{i(+)}^{+} = {}_{65}^{\infty}DV9_{i(+)}^{+}\right)$. This fits DV, while requiring certain totals be met. The first condition after the pipe requires the age and sex specific totals be the same as those in $DV9$; aside from that, the values are free to vary by region and special population. The second says that the counts for ages 65 and above for each county should be the same as those in $DV9$. The other totals are free to vary. Together, the totals across all regions must match the target age and sex specific totals, and the county-specific counts age 65+ must match the target.

The procedure can also be written as an econometric model: a log-linear model with offset. That notation is not included here. However, it reinforces that IPF is not just a trick to make totals match. Under certain conditions, it can actually be a type of conditional maximum likelihood fit.

As final notes, IPF does not work with negative values or excessive zeros. Therefore, such values should not be included in the input parameters.

Chapter 6
Deaths

One way for people to leave a population is through death. Deaths are projected first in UDEM, because only those who have survived are considered at risk to migrate or give birth. UDEM takes age-specific death rates for a region, applies them to the starting population, and outputs a number of deaths by age and sex. During the postcensal estimates period, the numbers are further adjusted to be consistent with published death counts, as discussed in Chapter 5. The county deaths are adjusted to match state totals. The modeled number of deaths is then subtracted from the population.

6.1 Death Computation

We begin with the "launch" or "starting" population at the beginning of the projection interval, ${}^{1}_{x}P0^{s}_{i(+)}$. The goal is to obtain the number of deaths, and also a "survived population." The procedure is the same for states and counties, except that counties require additional adjustment to state totals. The equations are generally presented for the counties, and they are easily adapted for the state.

Note that the death rates are applied to both subpopulations, but the deaths are removed from the general population (so that the special population can remain constant). This is not to say deaths did not occur among the special population, but that the accounting is simplified. The results are not affected, but interpretation requires the deaths to be considered for the full population (and not just the general).

6.1.1 Initial Death Calculation

Calculating the flow of deaths requires exogenous projections of death rates by single-year-of-age, region, and sex, ${}^{1}_{x}asdr0^{s}_{i(+)}$. In practice, the same rates can be used for the different strata, but the model requires them in this format. The rates are then multiplied by the starting population ${}_{x}^{1}P0_{i(+)}^{s}$ to obtain an initial deaths estimate, ${}_{x}^{1}D0_{i(+)}^{s}$.

$$ {}_{x}^{1}D0_{i(+)}^{s} = {}_{x}^{1}asdr0_{i(+)}^{s} * {}_{x}^{1}P0_{i(+)}^{s} \tag{6.1}$$

6.1.2 Adjusting Deaths to Exogenous Totals

For the postcensal years, an external estimate of deaths ${}_{0}^{\infty}DX_{i(+)}^{+}$ must be used for a control total. UDEM uses the estimates published by UPC.

Raking can be proportional or uniform. Define which raking type to use: proportionate or uniform for deaths. Which to use is stored as an indicator variable:$adu = 1$ for uniform, 0 for proportionate. Calculate the appropriate adjustment factor, afd and adjusted deaths, $D1$. The adjustment factor is useful for future model iterations or as a post-processing metric.

$$ {}_{0}^{\infty}afd_{i(+)}^{+} = \begin{cases} rake_p({}_{x}^{1}D0_{i(+)}^{s}, {}_{0}^{\infty}DX_{i(+)}^{+}, TRUE) \text{ if } adu = 0 \\ rake_u({}_{x}^{1}D0_{i(+)}^{s}, {}_{0}^{\infty}DX_{i(+)}^{+}, TRUE) \text{ if } adu = 1 \end{cases} \tag{6.2}$$

Also, get the adjusted values.

$$ {}_{x}^{1}D1_{i(+)}^{s} = \begin{cases} rake_p({}_{x}^{1}D0_{i(+)}^{s}, {}_{0}^{\infty}DX_{i(+)}^{+}, FALSE) \text{ if } adu = 0 \\ rake_u({}_{x}^{1}D0_{i(+)}^{s}, {}_{0}^{\infty}DX_{i(+)}^{+}, FALSE) \text{ if } adu = 1 \end{cases} \tag{6.3}$$

6.1.3 Adjusting County to State Totals

For the counties, these values must be controlled to state totals. This is true during the postcensal and projection periods. At the state level, no additional adjustment is needed.

$$ {}_{x}^{1}D_{+(+)}^{s} = {}_{x}^{1}D1_{+(+)}^{s} \tag{6.4}$$

The sums for the counties are required to match the totals from the state projection by age and sex. To adjust the county values to the state totals, the IPF technique is utilized, with the deaths by county, age, and sex as the seeds, and the new marginal totals being the state total deaths by age and sex.

$$ {}_{x}^{1}D_{i(+)}^{s} = IPF\left(seed = {}_{x}^{1}D1_{i(+)}^{s} | {}_{x}^{1}D_{+(+)}^{s} = {}_{x}^{1}D1_{+(+)}^{s}\right) \tag{6.5}$$

This procedure indicates the final deaths by age and sex are the deaths obtained by predicting from the previously-adjusted deaths, conditional on the new totals.

6.1.4 Adjusted Death Rates

At this point, it is useful to calculate adjusted death rates consistent with the adjusted deaths for use in post-processing and assessment. The values can be used to calculate measures such as adjusted life expectancy. The adjusted death rates are calculated by dividing the final deaths by the starting population.

$$ {}_{x}^{1}asdr_{i(+)}^{s} = \frac{{}_{x}^{1}D_{i(+)}^{s}}{{}_{x}^{1}P0_{i(+)}^{s}} \tag{6.6} $$

6.2 Survived Population

The survived general population is obtained by subtracting the deaths from the starting general population. The survived special population remains unchanged.

$$ {}_{x}^{1}PSURV_{i(k)}^{s} = \begin{cases} {}_{x}^{1}P0_{i(1)}^{s} - {}_{x}^{1}D_{i(+)}^{s} & \text{if } k = 1 \\ {}_{x}^{1}P0_{i(2)}^{s} & \text{if } k = 2 \end{cases} \tag{6.7} $$

Chapter 7
Births

New people can enter a population through birth. After accounting for deaths, UDEM takes projected age-specific birth rates as inputs, and applies them to the survived population of women during the middle of the year to produce a number of births. 48.8 percent of the births are assumed to be female, and the remainder male. As was done with deaths, the births are further adjusted if additional published birth estimates are available. Also, the county births are adjusted to be consistent with state numbers. The births are not added to the population until the "aging" process. Technical details follow.

7.1 Birth Computation

The procedure is the same for states and counties, except that counties require additional adjustment to state totals. The equations are generally presented for the counties, and they are easily adapted for the state.

Note that the birth rates are applied to both subpopulations, but the births are added to the general population (so that the special population can remain constant). The implications are similar to those discussed for deaths in the previous chapter.

7.1.1 Initial Birth Calculation

Calculating births requires exogenous projections of age-specific birth rates (for women) by single-year-of-age, and region, ${}^{1}_{x}asbr0^{2}_{i(+)}$. The birthrates are only applied to female population, and so the rates are not defined for males. In practice, the same rates can be used for the different strata, but the model requires them in this format. The rates are then multiplied by a "midpoint survived population," *PMID* to obtain an initial births estimate. The midyear survived population is utilized, because it is assumed half the deaths to childbearing-age women occurred during the first half of the projection interval. Note that for the births, the sex refers to the sex of child, but the age is the age of the mother. However, the population at risk of giving birth is the population of women at midpoint, so female is the only sex.

The midyear population is denoted ${}_{x}^{1}PMID_{i(s)}^{k}$, and calculated

$$ {}_{x}^{1}PMID_{i(k)}^{s} = \frac{{}_{x}^{1}P0_{i(k)}^{s} + {}_{x}^{1}PSURV_{i(k)}^{s}}{2} \tag{7.1} $$

Multiply the midyear female population by the age-specific birth rates to yield an initial births estimate by age of mother.

$$ {}_{x}^{1}B0_{i(+)}^{+} = {}_{x}^{1}asbr0_{i(+)}^{2} * {}_{x}^{1}PMID_{i(+)}^{2} \tag{7.2} $$

Now, the infants can be designated male or female. The ratio of male-to-female births (called the secondary sex ratio), is assumed to be constant at 1.05. This is the same as assuming approximately 48.8 percent of the births are female, and the remainder male.

$$ {}_{x}^{1}B0_{i(+)}^{s} = \begin{cases} \frac{1.05}{2.05} * {}_{x}^{1}B0_{i(+)}^{+} & \text{if } s = 1 \\ \frac{1.00}{2.05} * {}_{x}^{1}B0_{i(+)}^{+} & \text{if } s = 2 \end{cases} \tag{7.3} $$

7.1.2 Adjusting Birth to Exogenous Totals

For the postcensal years, and external estimate of births ${}_{0}^{\infty}BX_{i(+)}^{+}$ should be used for a control total.

Raking can be proportional or uniform. Define which raking type to use: proportionate or uniform for births. Which to use is stored as an indicator variable:$abu = 1$ for uniform, 0 for proportionate. Calculate the appropriate adjustment factor and adjusted values for each sex of child. The adjustment factor is useful for future model iterations or as a post-processing metric.

$$ {}_{0}^{\infty}afb_{i(+)}^{+} = \begin{cases} rake_{p}({}_{x}^{1}B0_{i(+)}^{s}, {}_{0}^{\infty}BX_{i(+)}^{+}, TRUE) \text{ if } abu = 0 \\ rake_{u}({}_{x}^{1}B0_{i(+)}^{s}, {}_{0}^{\infty}BX_{i(+)}^{+}, TRUE) \text{ if } abu = 1 \end{cases} \tag{7.4} $$

Also, get the adjusted values.

$$ {}_{x}^{1}B1_{i(+)}^{+} = \begin{cases} rake_{p}({}_{x}^{1}B0_{i(+)}^{s}, {}_{0}^{\infty}BX_{i(+)}^{+}, FALSE) \text{ if } abu = 0 \\ rake_{u}({}_{x}^{1}B0_{i(+)}^{s}, {}_{0}^{\infty}BX_{i(+)}^{+}, FALSE) \text{ if } abu = 1 \end{cases} \tag{7.5} $$

7.1.3 Adjusting County to State Totals

For the counties, these values must be controlled to state totals. This is true during the postcensal and projection periods. But, at the state level, no additional adjustment is needed:

$$ {}^{1}_{x}B^{+}_{+(+)} = {}^{1}_{x}B1^{+}_{+(+)} \tag{7.6} $$

The sums for the counties are required to match the state total by age and sex. To adjust the county values to the state totals, the IPF technique is utilized, with the births by county, age of mother, and sex as the seeds, and the new marginal totals being the state total births by age of mother and sex of child.

$$ {}^{1}_{x}B^{s}_{i(+)} = IPF\left(seed = {}^{1}_{x}B1^{s}_{i(+)} | {}^{1}_{x}B^{s}_{+(+)} = {}^{1}_{x}B1^{s}_{+(+)} \right) \tag{7.7} $$

This procedure indicates the final births by age are the births obtained by predicting from the previously-adjusted births, conditional on the new totals.

This is the final births variable. The births are added to the population during the aging process (becoming the new population aged 0 to 1).

At this point, it is useful to calculate adjusted birth rates consistent with the adjusted births for use in post-processing and assessment. The values can be used to calculate measures such as adjusted total fertility rates. The adjusted birth rates are calculated by dividing the final births by the starting population.

$$ {}^{1}_{x}asbr^{2}_{i(+)} = \frac{{}^{1}_{x}B^{+}_{i(+)}}{{}^{1}_{x}PMID^{2}_{i(+)}} \tag{7.8} $$

Chapter 8
Aging

While this section is short, it is one of the most important parts of the UDEM model. Every year, everyone that survives gets one year older, and the new births form the group aged 0. This helps the model be consistent with a population's history by tracking age waves of generations over time (for example, Baby Boomers or Millenials). The result is called the "aged and survived" population.

8.1 Aging Process

Here, everyone in the survived general population becomes one year older. Those survived individuals already age 100+ stay age 100+. The new births become those aged 0-1. The new population generated is called the "aged and Survived Population," denoted ${}_{x}^{1}PAGE_{i(k)}^{s}$. Only the general population is aged. The special population is identical to the starting population.

$$
{}_{x}^{1}PAGE_{i(k)}^{s} = \begin{cases} {}_{0}^{\infty}B_{i(1)}^{s} & \text{if } k=1, x=0 \\ {}_{x-1}^{1}PSURV_{i(1)}^{s} & \text{if } k=1, 1 \leq x \leq 99 \\ {}_{100}^{\infty}PSURV_{i(1)}^{s} + {}_{99}^{1}PSURV_{i(1)}^{s} & \text{if } k=1, x=100 \\ {}_{x}^{1}PSURV_{i(2)}^{s} & \text{if } k=2 \end{cases} \tag{8.1}
$$

It is important to note that deaths are not calculated for the new births. Rather, they get calculated during the following projection interval. This means deaths to infants in the current projection interval were actually from the last interval. Since most infant deaths occur within the first few days of life, this could introduce some bias. However, as the number of infant deaths does not vary considerably between two consecutive projection intervals, and infants have no direct effect upon the birth or net migrant calculations, we consider this bias to be trivial.

The main benefit of this approach is a cleaner model where the deaths are all calculated, and then all the births. We need not jump back and forth, which, given the adjustments of county to state totals, would considerably complicate the model.

Chapter 9
Migration

Economic forces induce migration in small areas. Accordingly, UDEM ties net migration to economic data, after accounting for births and deaths. Because the economic-demographic relationships are somewhat complex, many additional data points must be leveraged. The order of calculations changes depending upon data availability.

Projected rates of labor force participation and unemployment are input, and UDEM applies them to the aged and survived population to determine a labor force. Past relationships between jobs and population are then used to determine "maintenance employment," or the number of jobs needed to satisfy the population's demands. Another input is total employment, which UDEM then compares to the supply of labor. If there are more jobs than needed, people migrate in to work those jobs. The opposite can also occur. Dependents migrate with the laborers. Age and sex for net migrants are based upon past patterns.

Labor migration is only one type considered by UDEM. It also considers retirement, missionary, and student migration. The first two are explicitly modeled. Student migration is embedded in other calculations, because of a lack of data. Additional old-age labor force adjustments are made in counties with high levels of retirement migration. As with births and deaths, net migration numbers are adjusted to match published estimates, and county values must be consistent with the state.

9.1 Migration Overview

Three types of migration are specifically applied in the model: (1) retirement, (2) labor-tied, and (3) missionary. The first applies to individuals age 65+, the second to individuals under the age of 65 (including students) excepting missionaries, and the third to missionaries aged 18 through 24. Note the labor-tied and missionary migrations are modeled as exhaustive and mutually exclusive categories for migration to individuals aged 0 through 64.

Missionary migration is a peculiar feature of Utah's migratory patterns. While missionaries absent from Utah may in some sense be considered residents of Utah, the Census Bureau does not consider them to be so. This accounts for the distinctive "dips" or "caves" historically present around ages 18 through 24 in Utah's population pyramids at the decennial censuses, especially for males. Thus, they are not considered as "resident population" in UDEM. However, there is a separate accounting to model traditional patterns of "seniority" and return times. To address various data issues, missionary migration is only directly modeled at the state level, and when county migration patterns are statistically controlled to state totals, the IPF procedure allocates the state-level missionary migrants proportionately amongst the counties.

Retirement migration is only specifically projected at the state-level via old-age dependency ratios. Migration for the counties is then determined by an allocation formula using values obtained from net migration patterns observed between the 2000 and 2010 decennial censuses, with age and sex patterns then raked proportionately to the interval's beginning age and sex structure.

Labor-tied migration is determined with a combination of projected employment (jobs), labor-force-participation rates, unemployment rates, and the ratio of jobs to employed persons. The same procedure is used at the state and county levels, though the county results must then be statistically controlled to the state totals. Note the student migration, as well as child migration, are assumed to be tied to employment, with child migrants in particular being tied to labor migrants by the childhood dependency ratios. (Note that the unique demographic signatures for student counties are maintained through the county special population procedures).

The order of the calculations changes depending upon data availability. We begin by first delineating the overall migration model logic and key parameters. The different subprocesses that can be used to calculate the various parameters (depending again upon data availability) are then presented separately.

Note that during the postcensal estimates period, the model requires county-level estimates of the population aged 65+, which can be obtained from Medicare records. It also requires estimates of total net migration. From these points of data, retirement migration and labor-tied migration are directly inferred. Estimated or projected gross missionary out-migration rates must be input. When combined with assumptions of missionary service time, these model missionary migration flows. At the state level, all the migration types must be projected. At the county level, the 65+ migration is down-allocated from the state values. The only migration explicitly modeled at the county level (during the projections period) is labor-tied migration.

9.2 Main Migration Procedure

The goal is to go from the aged and survived population ${}_{x}^{1}PAGE_{i(k)}^{s}$ to the final population (i.e., the starting population for the next period), denoted by ${}_{x}^{1}P_{i(k)}^{s}$. As always, special populations remain unchanged. For the general population, net migration must be modeled.

The following identities must hold:

$$ {}_{x}^{1}P_{i(k)}^{s} = \begin{cases} {}_{x}^{1}PAGE_{i(1)}^{s} + {}_{x}^{1}M_{i(1)}^{s} & \text{if } k = 1 \\ {}_{x}^{1}PAGE_{i(2)}^{s} & \text{if } k = 2 \end{cases} \tag{9.1} $$

$$ {}_{x}^{1}M_{i(1)}^{s} = {}_{x}^{1}LTM_{i(1)}^{s} + {}_{x}^{1}RM_{i(1)}^{s} + {}_{x}^{1}HM_{i(1)}^{s} \tag{9.2} $$

In the latter equation, LTM, RM, and HM are labor-tied, retirement, and missionary net migrations, respectively. The three net migrations sum to the total net migration M along each dimension.

9.3 Common Procedures

Certain procedures are often repeated for the migration component, and they are included here as function calls. These common procedures relate to estimating "jobs-to-employed" at the end of each interval, and missionary accounting.

9.3.1 End of Interval Jobs-to-employed Estimation

The population of working age PWA (which is 16 through 80 in UDEM, though this can be adjusted), is multiplied by labor force participation rates and employment rates to yield an employed labor force. Dividing this into the number of jobs yields the region-specific jpe measure, which is assumed constant across age, sex, and subpopulation.

The necessary inputs are an aged and survived population, $PAGE$, employment, $JOBS$, labor-force participation rates, $lfpr$, and employment rates, $empr$. While only the jpe needs to be returned, other variables are also generated, and can be useful for other analyses. These include labor force, LF, employed labor force, ELF, and population of working age, PWA, for the current projection interval. Note that in the latest UDEM projections, labor force participation rates and unemployment rates for the entire state were used for the entire county, and the JPE measure captured any county-specific deviations from those values. However, the model is designed in such as way to permit those rates to vary explicitly for each county.

The function call is: $POSTJPE\left({}_{x}^{1}P_{i(+)}^{s}, {}_{x}^{1}lfpr_{i(+)}^{s}, {}_{0}^{\infty}empr_{i(+)}^{+}, {}_{0}^{\infty}JOBS_{i(+)}^{+}\right)$

$$ {}_{x}^{1}PWA_{i(+)}^{s} = {}_{x}^{1}P_{i(+)}^{s} \text{ where } 16 \leq x \leq 80 \quad (9.3)$$

$$ {}_{x}^{1}LF_{i(+)}^{s} = {}_{x}^{1}lfpr_{i(+)}^{s} * {}_{x}^{1}PWA_{i(+)}^{s} \quad (9.4)$$

$$ {}_{x}^{1}ELF_{i(+)}^{s} = {}_{0}^{\infty}empr_{i(+)}^{+} * {}_{x}^{1}LF_{i(+)}^{s} \quad (9.5)$$

$$ {}_{0}^{\infty}jpe_{i(+)}^{+} = \frac{{}_{0}^{\infty}JOBS_{i(+)}^{+}}{{}_{0}^{\infty}ELF_{i(+)}^{s}} \quad (9.6)$$

9.3.1.1 Retirement Jobs-to-employed Adjustment

The population aged 65+ can still participate in the labor force. In the current version of UDEM, labor force participation rates, $lfpr$, apply up through age 80. As the retired portion of a population changes, by definition the labor force participation rates should also change. It is assumed that the state-level rates account for this. However, the county-level $lfpr$'s were proportionately scaled by the jobs-to-employed (jpe) across the the entire age and sex schedule of $lfpr$'s. This means no additional adjustment must necessarily be made beforehand to account for the county-specific linkage between $lfpr$ and retirement migration.

To account for important county-retirement patterns, UDEM can estimate a different jpe term for retirement ages 65+. This is achieved by estimating an additional multiplicative factor for the age 65+ population during the postcensal estimates period, and then applying that factor during the projections period, as detailed below.

The estimation procedure follows three steps during each projection interval. (1) calculate the aged (65+) labor force in the absence of retirement migration, ${}_{x}^{1}ALF0_{i(+)}^{s}$ (2) calculate the labor force after retirement migration, ${}_{x}^{1}ALF_{i(+)}^{s}$, (3) calculate the ratio as a post-migration adjustment factor, ${}_{65}^{\infty}postalfa_{i(+)}^{+}$.

The function call is: $POSTALFA\left({}_{x}^{1}PAGE_{i(+)}^{s}, {}_{x}^{1}P_{i(+)}^{s}, {}_{x}^{1}lfpr_{i(+)}^{s},\right)$

$$ {}_{x}^{1}ALF0_{i(+)}^{s} = {}_{x}^{1}lfpr_{i(+)}^{s} * {}_{x}^{1}PAGE_{i(+)}^{s} \text{ where } x \geq 65 \quad (9.7)$$

$$ {}_{x}^{1}ALF_{i(+)}^{s} = {}_{x}^{1}lfpr_{i(+)}^{s} * {}_{x}^{1}P_{i(+)}^{s} \text{ where } x \geq 65 \quad (9.8)$$

$$ {}_{65}^{\infty}postalfa_{i(+)}^{+} = \frac{{}_{x}^{1}ALF0_{i(+)}^{s}}{{}_{x}^{1}ALF_{i(+)}^{s}} \quad (9.9)$$

If the labor force after retirement migration is larger than prior, then this value will be less than 1. When multiplied by the $lfpr$ for the age 65+ population, the $lfpr$ will be adjusted so that those migrants are not in the labor force. A key assumption is that none of the age 65+ migrants will be in the labor force. This is reasonable, given the vast demographic literature that treats age 65+ migration as retirement-related. Note these values are estimated throughout the estimates period, and that average is applied into the future.

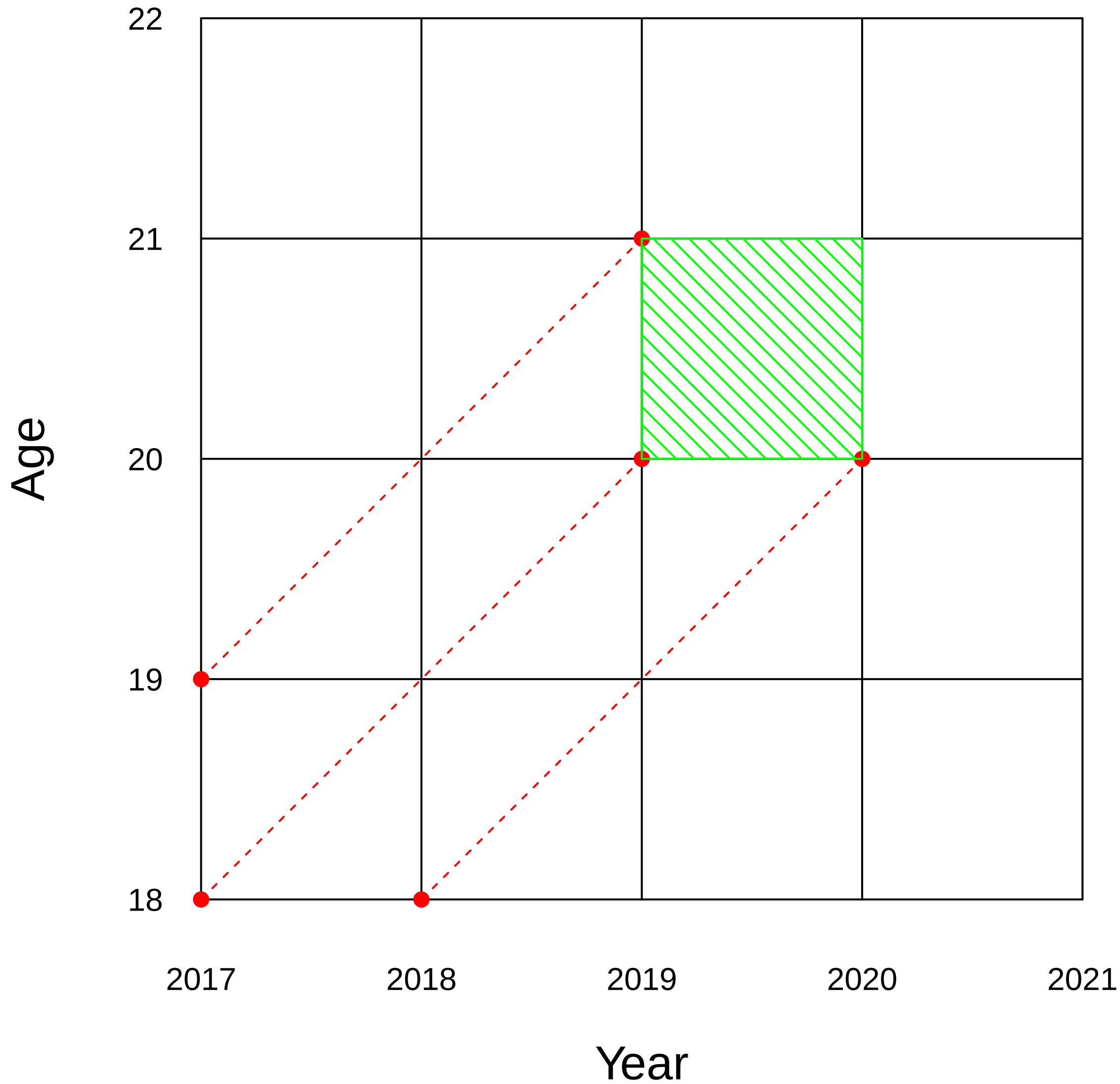
9.3.2 Missionary Migration

Net missionary migration is the difference between returning (in) and departing (out) missionaries. Due to a peculiar history in regards to the missionary accounting, the model often identifies them as "heralds." Hence, many of the variables have an "H" in them.

The departing missionaries are assumed to leave from the general population (note that in practice, this permits transfers between special populations and the general population as long as the values "wash out" in the end). They are denoted $HERD$. Similarly, the returning missionaries are assumed to return to the general population. They are denoted $HERR$.

Rates are used to calculate departing missionaries, and normal service times are used to calculate returning missionaries. Figure 9.1 is an example Lexis diagram for our assumed typical male service ages and times.

Fig. 9.1 Male Missionary Lexis Diagram



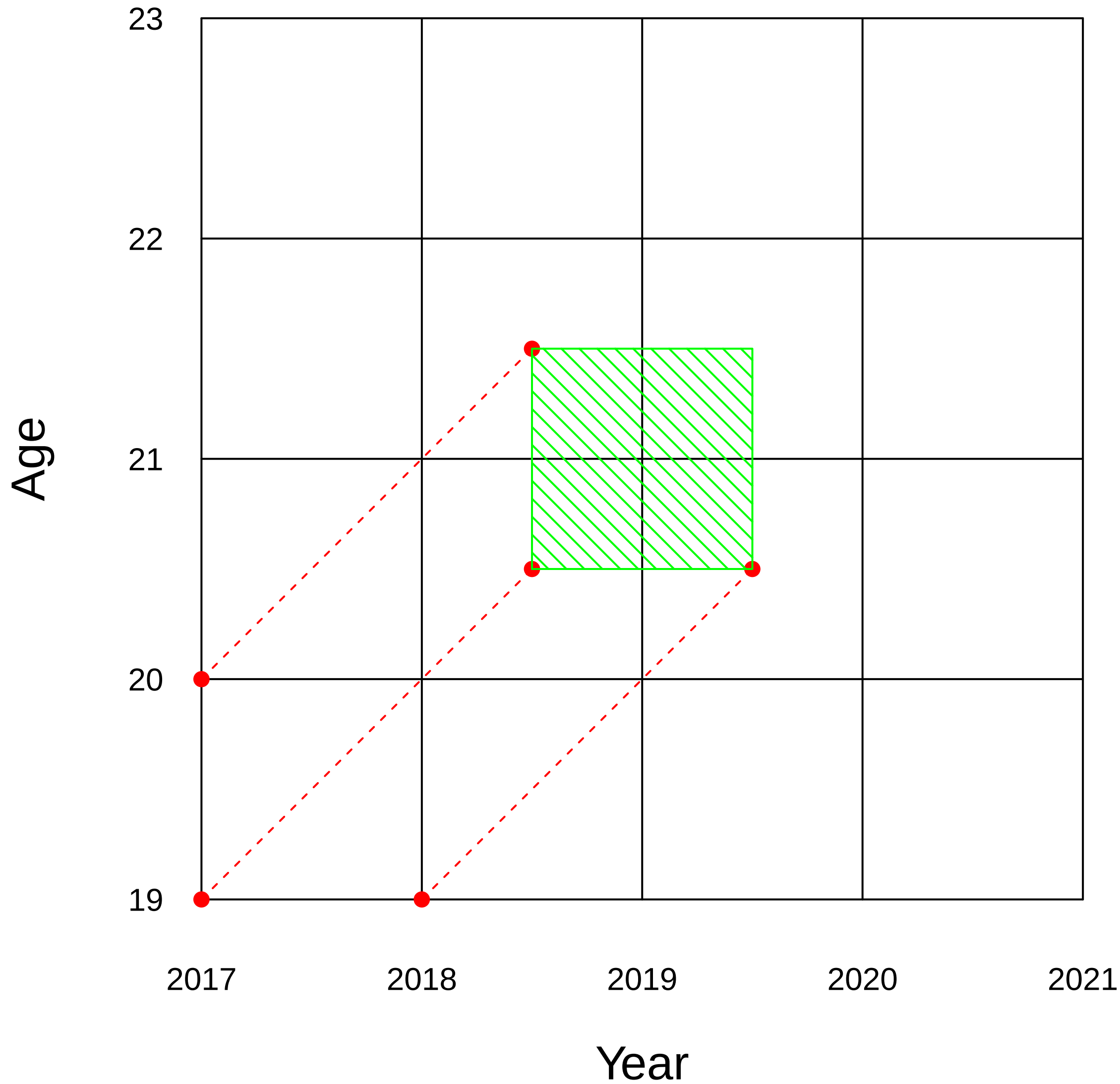
Source: Kem C. Gardner Policy Institute

The diagram shows the departing and returning times for male missionaries that departed aged 18-19 during 2017-2018. The red dots are the limiting ages and years of departure and return. The dashed diagonal lines represent the respective cohorts. The shaded box shows the ages and years of the returning missionaries. We assume all male service times are exactly two years, and therefore all these missionaries would return aged 20-21 during 2019-2020.

Missionaries need not be aged 18-19 at departure, and the years will vary. So, a more general algorithm is that male missionaries departing aged x to $x+1$ between years t and $t+1$ would return aged $x+2$ to $x+3$ between years $t+2$ and $t+3$.

Figure 9.2 is a comparable diagram for female missionaries.

Fig. 9.2 Female Missionary Lexis Diagram



Source: Kem C. Gardner Policy Institute

Notice that females are assumed to serve exactly 1.5 years, which complicates the algorithm. We assume the departing missionaries are jointly uniformly distributed within the age-year square. Then, for female missionaries departing aged x to $x+1$ between years t and $t+1$, the returning missionaries can be evenly divided among these four groups:

- Aged $x+1$ to $x+2$ between year $t+1$ and $t+2$
- Aged $x+1$ to $x+2$ between year $t+2$ and $t+3$
- Aged $x+2$ to $x+3$ between year $t+1$ and $t+2$
- Aged $x+2$ to $x+3$ between year $t+2$ and $t+3$

UDEM accounts for this by setting up a separate stock of missionaries, and tracking them over time. The missionary stock is denoted by $HERST$. The subscript for special populations is unnecessary among missionary stocks, but seniority is important. Therefore, the usual special population subscript k is replaced by a subscript for seniority, g, with 1 representing first-year missionaries and 2 second-year.

The Utah resident population is assumed to be aged prior to the missionary migration. Therefore, the missionary stock must still be aged, and this is why $x-1$ appears in the equations.

Rates are used to determine departing missionaries. The out-going rates are encapsulated in the variable, ${}_{x}^{1}hdr_{+(1)}^{s}$.

$$ {}_{x}^{1}hdr_{+(1)}^{s} = \frac{{}_{x}^{1}HERD_{+(1)}^{s}}{{}_{x}^{1}PAGE_{+(1)}^{s}} \tag{9.10} $$

These rates are applied to the general population to produce the necessary missionary migration flows.

The function call is $HCALCP\left({}_{x}^{1}PAGE_{+(1)}^{s}, {}_{x}^{1}hdr_{+(1)}^{s}, {}_{x}^{1}HERSTO_{+(g)}^{s}\right)$

To apply the rates, use:

$$ {}_{x}^{1}HERD_{+(1)}^{s} = {}_{x}^{1}hdr_{+(1)}^{s} * {}_{x}^{1}PAGE_{+(1)}^{s} \tag{9.11} $$

Returning missionaries are calculated directly from the old missionary stock. It is assumed there is no mortality or fertility among the missionaries, and that all the departing missionaries return to Utah. It is assumed the males return after two years, and females after 1.5 years. Note the following equations assume the starting missionary stock $HERSTO$ has already been modeled.

$$ {}_{x}^{1}HERR_{+(1)}^{s} = \begin{cases} {}_{x}^{1}HERSTO_{+(2)}^{1} & \text{if s=1} \\ {}_{x}^{1}HERSTO_{+(2)}^{2} + \left(\frac{1}{2}\right) {}_{x}^{1}HERSTO_{+(1)}^{2} & \text{if s=2} \end{cases} \tag{9.12} $$

The missionary stock is then "aged" from the "old" $HERSTO$ stock to the "new" $HERST$ stock as follows:

$$
{}^{1}_{x}HERST^{s}_{+(g)} = \begin{cases} {}^{1}_{x}HERD^{1}_{+(1)} & \text{if g=1 and s=1} \\ {}^{1}_{x}HERD^{2}_{+(1)} & \text{if g=1 and s=2} \\ {}^{1}_{x-1}HERSTO^{1}_{+(1)} & \text{if g=2 and s=1} \\ \left(\frac{1}{2}\right) * {}^{1}_{x-1}HERSTO^{2}_{+(2)} & \text{if g=2 and s=2} \end{cases} \tag{9.13}
$$

The net missionary migration is the returning minus the departing, and this is the value that is returned. But, the other values may be used in future projection intervals.

$$
{}^{1}_{x}HM^{s}_{+(1)} = {}^{1}_{x}HERR^{s}_{+(1)} - {}^{1}_{x}HERD^{s}_{+(1)} \tag{9.14}
$$

9.4 Preliminary Data Setup

Some frequently-used variables can be set up beforehand. The most useful are dependency ratios. The ratio of population under 18 to the population 19 through 64 is called the "childhood dependency ratio", cdr. It is used to calculate the number of children who migrate with laborers. The "aged dependency ratio," adr, is used similarly, though only at the state level, and the interpretation is slightly different. Instead of being assumed to migrate with the laborers, it is assumed that retirement migrants for the state as a whole must be consistent with the state's economic carrying capacity. The total dependency ratio, dr, is their sum.

$$
{}^{\infty}_{0}adr^{+}_{+(k)} = \frac{{}^{\infty}_{65}P0^{+}_{+(k)}}{{}^{46}_{18}P0^{+}_{+(k)}} \tag{9.15}
$$

$$
{}^{\infty}_{0}cdr^{+}_{+(k)} = \frac{{}^{17}_{0}P0^{+}_{+(k)}}{{}^{46}_{18}P0^{+}_{+(k)}} \tag{9.16}
$$

$$
{}^{\infty}_{0}dr^{+}_{+(k)} = {}^{\infty}_{0}adr^{+}_{+(k)} + {}^{\infty}_{0}cdr^{+}_{+(k)} \tag{9.17}
$$

The same apply at the county and state levels. To eliminate confusion using the demographic notation, note that 18 + 46 = 64.

9.5 State-level Net Migration

The state-level migration differs by whether or not postcensal estimates, a "target" estimate, or no estimate of migration flows are available. The "target" may be provided for scenario work; or to address the "jump-off" bias that occurs when moving from an estimates to projections period, or to help the net migration component better integrate as the economic short-term cycle forecasts merge with the trend.

Although both the “estimates” and “target” methods provide a control total for net migration, during the “estimates” period additional information (e.g., stocks estimated from Medicare tabulations) can inform the age 65+ population. The methods can be adapted judiciously to accommodate varying data availability.

9.5.1 State-level Net Migration with Postcensal Estimates

When the estimate MX is available the total net migration will equal the estimate:

$$ {}_{0}^{\infty}M_{+(1)}^{+} = {}_{0}^{\infty}MX_{+(1)}^{+} \tag{9.18}$$

Next, the missionary migration is calculated.

$$ {}_{x}^{1}HM_{+(1)}^{s} = HCALCP\left({}_{x}^{1}PAGE_{+(1)}^{s}, {}_{x}^{1}hdr_{+(1)}^{s}, {}_{x}^{1}HERSTO_{+(g)}^{s}\right) \tag{9.19}$$

The “non-missionary” net migration is the residual:

$$ {}_{0}^{\infty}NHM_{+(1)}^{+} = {}_{0}^{\infty}M_{+(1)}^{+} - {}_{0}^{\infty}HM_{+(1)}^{+} \tag{9.20}$$

An external estimate of the age 65+ population, ${}_{0}^{\infty}PO65_{+(+)}^{+}$, should also be available from external sources. The residual between that estimate and the survived and aged 65+ population is the total retirement net migration, RM. (Note that there are no special populations past age 65, and therefore ${}_{0}^{\infty}PO65_{+(1)}^{+} = {}_{0}^{\infty}PO65_{+(+)}^{+}$.)

$$ {}_{0}^{\infty}RM_{+(1)}^{+} = {}_{0}^{\infty}PO65_{+(1)}^{+} - {}_{0}^{\infty}PAGE_{+(1)}^{+} \text{ where x} \geq 65 \tag{9.21}$$

Based upon the identities, the total net migration is the sum of labor-tied, missionary, and retirement. So, labor-tied migration can be calculated as a residual:

$$ {}_{0}^{\infty}LTM_{+(1)}^{+} = {}_{0}^{\infty}NHM_{+(1)}^{+} - {}_{0}^{\infty}RM_{+(1)}^{+} \tag{9.22}$$

The labor-tied migration consists of migrants of working age, along with dependents. To graduate labor-tied net migration by age, net migration age schedules for ages 0 through 64 are utilized. First, an “expected” net migration is calculated by applying this schedule of rates to the starting general population.

$$ {}_{x}^{1}XLTM_{+(1)}^{+} = {}_{x}^{1}P0_{+(1)}^{+} * {}_{x}^{1}nmsched0_{+(1)}^{+} \text{ where } 0 \leq x \leq 64 \tag{9.23}$$

The target schedule can then be adjusted through the uniform raking procedure to match the labor-tied migration already calculated. Alternatively, this can be viewed as distributing the difference in totals uniformly and adding it to the expected age distribution. The uniform procedure is more appropriate here, because negative values are expected, and the proportionate procedure will yield results incompatible with dependency ratios.

$$ {}_{x}^{1}LTM_{+(1)}^{+} = rake_{u}\left({}_{x}^{1}XLTM_{+(1)}^{+}, {}_{0}^{\infty}LTM_{+(1)}^{+}, FALSE, FALSE\right) \tag{9.24}$$

Inspecting the net migration patterns for Utah and its counties between 2000 and 2010 did not suggest sex patterns for non-missionary and non-retirement migration were different enough to justify including additional sex-specific parameters. Therefore, it was simply assumed half of the net migrants were female, the other half male. This means the count of net migrants by age and sex is identical for males and females. (The proportions can be easily adjusted as required).

$$ {}_{x}^{1}LTM_{+(1)}^{s} = \begin{cases} \left(\frac{1}{2}\right){}_{x}^{1}LTM_{+(1)}^{+} & \text{if s=1} \\ \left(\frac{1}{2}\right){}_{x}^{1}LTM_{+(1)}^{+} & \text{if s=2} \end{cases} \tag{9.25}$$

At this point, missionary and labor-tied net migration are graduated by age and sex. Retirement migration still must be graduated. It is assumed the retirement net migrants are distributed proportionately to the age-sex distribution of the general launch population aged 65+.

$$ {}_{x}^{1}RM_{+(1)}^{s} = {}_{0}^{\infty}RM_{+(1)}^{+} * \frac{{}_{x}^{1}P0_{s(1)}^{+}}{{}_{64}^{\infty}P0_{s(1)}^{+}} \text{where } x \geq 65 \tag{9.26}$$

The final state-level net migration and ending population can be calculated according to equations 9.1 and 9.2, adapted to the state level.

$$ {}_{x}^{1}P_{+(k)}^{s} = \begin{cases} {}_{x}^{1}PAGE_{+(1)}^{s} + {}_{x}^{1}M_{+(1)}^{s} & \text{if } k=1 \\ {}_{x}^{1}PAGE_{+(2)}^{s} & \text{if } k=2 \end{cases} \tag{9.27}$$

$$ {}_{x}^{1}M_{+(1)}^{s} = {}_{x}^{1}LTM_{+(1)}^{s} + {}_{x}^{1}RM_{+(1)}^{s} + {}_{x}^{1}HM_{+(1)}^{s} \tag{9.28}$$

Lastly, the jobs-per-employed can be estimated by calling the function detailed above,

$$ {}_{0}^{\infty}jpe_{+(+)}^{+} = POSTJPE\left({}_{x}^{1}P_{+(+)}^{s}, {}_{x}^{1}lfpr_{+(+)}^{s}, {}_{0}^{\infty}empr_{+(+)}^{+}, {}_{0}^{\infty}JOBS_{+(+)}^{+}\right) \tag{9.29}$$

Notice that the jobs were not used to estimate the net migration. Instead, the actual postcensal estimate of net migration was used to estimate the link between jobs and population, controlling for the labor force participation and employment rates. The jpe factor is then used in later projection intervals.

9.5.2 State-level Net Migration Projections with a Target

Sometimes, we do not have an estimate of net migration, but need to incorporate an external "target." For example, we often have short-term employment projections informed by knowledge of the business cycle; but, long-term projections must be based upon the trend. Linking the two proves very difficult in a forecast model. "Jump-off" bias often occurs, where minor incongruities between the most recent, short-term forecast, and eventual trend projected averages explode into non-trivial irregularities and unreasonable projections.

To address this issue, external forecasts of state-level net migration can be used as a "target" in the short-term. This target can be developed by "smoothing over" initial net migration forecasts for use in subsequent runs. The net migration target is denoted ${}_{0}^{\infty}MTARG_{+(1)}^{+}$, and the first step is to assign this value to the total net migration, ${}_{0}^{\infty}M_{+(1)}^{+}$.

$$ {}_{0}^{\infty}M_{+(1)}^{+} = {}_{0}^{\infty}MTARG_{+(1)}^{+} \tag{9.30} $$

Next, the missionary migration is calculated.

$$ {}_{x}^{1}HM_{+(1)}^{s} = HCALCP\left({}_{x}^{1}PAGE_{+(1)}^{s}, {}_{x}^{1}hdr_{+(1)}^{s}, {}_{x}^{1}HERSTO_{+(g)}^{s}\right) \tag{9.31} $$

Calculate the non-missionary migrants.

$$ {}_{0}^{\infty}NHM_{+(1)}^{+} = {}_{0}^{\infty}M_{+(1)}^{+} - {}_{0}^{\infty}HM_{+(1)}^{+} \tag{9.32} $$

We now need to know the number of "laboring labor-tied migrants," $LLTM$.

$$ {}_{0}^{\infty}LLTM_{+(1)}^{+} = \frac{{}_{0}^{\infty}NHM_{+(1)}^{+}}{1 + {}_{0}^{\infty}dr_{+(1)}^{+}} \tag{9.33} $$

This is the number of net migrants actually employed in the labor force. It is assumed that the number of retirement migrants is consistent with the economic carrying capacity added by the laboring migrants. This is captured by the aged dependency ratio, adr. Additionally, some children will migrate with laborers. This is captured by the childhood dependency ratio, cdr. So, with each laboring labor-tied migrant we assume an additional number of dependents (some younger, some older), that is measured by the total dependency ratio dr. In this way, dependency ratios can convert the non-heralding migrants into laboring labor-tied migrants, child dependents of labor migrants, and retirement migrants.

$$ {}_{0}^{\infty}RM_{+(1)}^{+} = {}_{0}^{\infty}LLTM_{+(1)}^{+} * {}_{0}^{\infty}adr_{+(1)}^{+} \tag{9.34} $$

$$ {}_{0}^{\infty}LTM_{+(1)}^{+} = {}_{0}^{\infty}NHM_{+(1)}^{+} - {}_{0}^{\infty}RM_{+(1)}^{+} \tag{9.35} $$

From this point onward, the process is identical to that used during the estimates period, using equations 9.23 - 9.29.

9.5.3 State-level Net Migration Unconstrained Projections

During the projections period, there are no external constraints on the model. First, project the missionary migration:

$$ {}_{x}^{1}HM_{+(1)}^{s} = HCALCP\left({}_{x}^{1}PAGE_{+(1)}^{s}, {}_{x}^{1}hdr_{+(1)}^{s}, {}_{x}^{1}HERSTO_{+(g)}^{s}\right) \quad (9.36) $$

The aged and survived post-missionary migration population $POSTH$ is now calculated. The missionary migrants are applied to the general population. The special population remains unchanged.

$$ {}_{x}^{1}POSTH_{+(k)}^{s} = \begin{cases} {}_{x}^{1}HM_{+(1)}^{s} + {}_{x}^{1}PAGE_{+(1)}^{s} & \text{if k=1} \\ {}_{x}^{1}PAGE_{+(2)}^{s} & \text{if k=2} \end{cases} \quad (9.37) $$

The population of working age must be calculated.

$$ {}_{x}^{1}PWA_{+(+)}^{s} = {}_{x}^{1}PAGE_{+(+)}^{s} \text{ where } 16 \leq x \leq 80 \quad (9.38) $$

Next, determine the labor force.

$$ {}_{x}^{1}LF_{+(+)}^{s} = {}_{x}^{1}PWA_{+(+)}^{s} * {}_{x}^{1}lfpr_{+(+)}^{s} \quad (9.39) $$

Apply employment rates to yield the employed labor force.

$$ {}_{x}^{1}ELF_{+(+)}^{s} = {}_{0}^{\infty}empr_{+(+)}^{+} * {}_{x}^{1}LF_{+(+)}^{s} \quad (9.40) $$

The number of "maintenance jobs," $JMAINT$, is required. This is the employment necessary to satisfy the already-present population. This value is obtained by multiplying the employed labor force times the jobs-per-employed factor.

$$ {}_{0}^{\infty}JMAINT_{+(+)}^{+} = {}_{0}^{\infty}jpe_{+(+)}^{+} * {}_{0}^{\infty}ELF_{+(+)}^{s} \quad (9.41) $$

There will be some difference between the maintenance jobs and the projected jobs, $JOBS$. That difference is $JMIG$, the jobs available for potential net migrants. Note that it can be positive or negative.

$$ {}_{0}^{\infty}JMIG_{+(+)}^{+} = {}_{0}^{\infty}JOBS_{+(+)}^{+} - {}_{0}^{\infty}JMAINT_{+(+)}^{+} \quad (9.42) $$

The number of jobs must again be scaled by the jobs-per-employed to determine the number of laboring labor migrants.

$$ {}_{0}^{\infty}LLTM_{+(+)}^{+} = \frac{{}_{0}^{\infty}JMIG_{+(+)}^{+}}{{}_{0}^{\infty}jpe_{+(+)}^{+}} \quad (9.43) $$

From this point onward, the procedure is identical to projections with a target, using equations 9.34, 9.35, and 9.23 - 9.29.

9.6 County-level Net Migration

At the county-level, the employment input *JOBS* must be adjusted to account for commuting patterns, since employees in one county can reside in a different county, confounding the jobs-to-person relationship. This is done through a "reverse commuting" matrix, *rcommr*. A commuting matrix quantifies the proportion of employees residing in a given county i that work in a county j. The reverse matrix quantifies the proportion of jobs in county j that are worked by residents of county i. Applying the matrix to the vector of jobs yields the commuting-adjusted jobs, *CJOBS*. This is calculated first.

$$ {}^{\infty}_{0}CJOBS^{+}_{i(+)} = \sum_{j=1}^{29} \left({}^{\infty}_{0}rcommr^{+}_{i,j(+)} * {}^{\infty}_{0}JOBS^{+}_{j(+)} \right) \tag{9.44} $$

After this calculation, the appropriate method depends upon whether or not a postcensal estimate is available.

9.6.1 County-level Net Migration with Postcensal Estimates

First, retirement migration can be calculated from the aged and survived population, and the exogenously-provided population aged 65+ *PO*65.

$$ {}^{\infty}_{0}RM^{+}_{i(1)} = {}^{\infty}_{65}PO65^{+}_{i(1)} - {}^{\infty}_{65}PAGE^{+}_{i(1)} \text{ where } x \geq 65 \tag{9.45} $$

The total net migration is also assigned the estimates target *MX*.

$$ {}^{\infty}_{0}M^{+}_{i(1)} = {}^{\infty}_{0}MX^{+}_{i(1)} \tag{9.46} $$

The residual migration can be assumed to be labor-tied. Generate the initial labor-tied migration total. (Note, it is "initial," because it will be adjusted when the county values are controlled to state totals).

$$ {}^{\infty}_{0}LTM0^{+}_{i(1)} = {}^{\infty}_{0}M^{+}_{i(1)} - {}^{\infty}_{0}RM^{+}_{i(1)} \tag{9.47} $$

Distribute the initial retirement migrants according to the starting general population, to obtain an initial estimate of retirement migration by age and sex.

$$ {}^{1}_{x}RM0^{s}_{i(1)} = {}^{\infty}_{0}RM^{+}_{i(1)} * \frac{{}^{1}_{x}PO^{+}_{i(1)}}{{}^{\infty}_{64}PO^{+}_{i(1)}} \text{ where } x \geq 65 \tag{9.48} $$

Predict the "expected" age schedule of labor-tied migrants, as was done for the state.

$$ {}_{x}^{1}XLTM_{i(1)}^{+} = {}_{x}^{1}P0_{i(1)}^{+} * {}_{x}^{1}nmsched0_{+(1)}^{+} \text{ where } 0 \le x \le 64 \tag{9.49}$$

Use the uniform raking procedure to adjust the expected values to the initial labor-tied migration total.

$$ {}_{x}^{1}LTM0_{i(1)}^{+} = rake_{u}\left({}_{x}^{1}XLTM_{i(1)}^{+}, {}_{0}^{\infty}LTM0_{i(1)}^{+}, FALSE, FALSE\right) \tag{9.50}$$

Assume half of the net migrants are female, and half male, at all ages.

$$ {}_{x}^{1}LTM0_{i(1)}^{s} = \begin{cases} \left(\frac{1}{2}\right) {}_{x}^{1}LTM0_{i(1)}^{+} & \text{if s=1} \\ \left(\frac{1}{2}\right) {}_{x}^{1}LTM0_{i(1)}^{+} & \text{if s=2} \end{cases} \tag{9.51}$$

As with county-level death and birth projections, the county-level net migration estimates are constrained to sum to state totals. Net migration presents an additional complication, because values can be negative. The IPF procedure does not work with negative values. This issue can be addressed by adding the initial net migration estimate to the survived and aged population to get an initial final population (which is always positive). The county-level populations can then be adjusted via IPF to match the state totals, and adjusted net migration then calculated as residual.

First, calculate the initial unadjusted population, separately for retirement and non-retirement aged populations.

$$ {}_{x}^{1}PUNJ_{i(1)}^{s} = \begin{cases} {}_{x}^{1}PAGE_{i(1)}^{s} + {}_{x}^{1}LTM0_{i(1)}^{s} & \text{if } x < 65 \\ {}_{x}^{1}PAGE_{i(1)}^{s} + {}_{x}^{1}RM0_{i(1)}^{s} & \text{if } x \ge 65 \end{cases} \tag{9.52}$$

Apply the IPF procedure to match the state totals. The starting seed values are the unadjusted population ${}_{x}^{1}PUNJ_{i(1)}^{s}$. For both retirement and non-retirement aged populations, the totals are constrained to match the state totals by age and sex. For retirement aged populations, the county totals are also constrained to be the county total already calculated. This is to maintain the apportionment of retirees.

$$ {}_{x}^{1}P_{i(k)}^{s} = \begin{cases} IPF\left(seed = {}_{x}^{1}PUNJ_{i(1)}^{s} | {}_{x}^{1}P_{+(1)}^{s} = {}_{x}^{1}P_{+(1)}^{s}\right) \\ \qquad \text{if } k = 1, x < 65 \\ IPF\left(seed = {}_{x}^{1}PUNJ_{i(1)}^{s} | {}_{x}^{1}P_{+(1)}^{s} = {}_{x}^{1}P_{+(1)}^{s}, {}_{65}^{\infty}P_{i(+)}^{+} = {}_{65}^{\infty}PO65_{i(+)}^{+}\right) \\ \qquad \text{if } k = 1, x \ge 65 \\ {}_{x}^{1}PAGE_{+(2)}^{s} \\ \qquad \text{if } k = 2 \end{cases} \tag{9.53}$$

Now, calculate the net migration as the residual for ages under 65 and 65+. The IPF procedure has adjusted the schedules in the missionary ages to be consistent with the state totals. So, LTM is not strictly LTM. It is the sum of two variables, LTM and HM. But, the two are not able to be separated within the model. This is not a perfect solution, but is a good approximation given the difficulties and complexity that would be introduced by explicitly modeling county-specific missionary migration patterns. We define a new variable for under 65 net migration, $U65M$, which assumes that:

$$ {}_{x}^{1}U65M_{i(1)}^{s} = {}_{x}^{1}LTM_{i(1)}^{s} + {}_{x}^{1}HM_{i(1)}^{s} \text{where x ¡ 65} \tag{9.54} $$

We can then calculate $U65M$:

$$ {}_{x}^{1}U65M_{i(1)}^{s} = {}_{x}^{1}P_{i(1)}^{s} - {}_{x}^{1}PAGE_{i(1)}^{s} \text{ where } x < 65 \tag{9.55} $$

$$ {}_{x}^{1}RM_{i(1)}^{s} = {}_{x}^{1}P_{i(1)}^{s} - {}_{x}^{1}PAGE_{i(1)}^{s} \text{ where } x \geq 65 \tag{9.56} $$

The sum of these migrants is the total migration. The missionary migration was handled along with labor-tied migration during the IPF procedure.

$$ {}_{x}^{1}M_{i(1)}^{s} = {}_{x}^{1}U65M_{i(1)}^{s} + {}_{x}^{1}RM_{i(1)}^{s} \tag{9.57} $$

Similar to the state-level model, the jobs-per-employed should be calculated.

$$ {}_{0}^{\infty}jpe_{i(+)}^{+} = POSTJPE\left({}_{x}^{1}P_{i(+)}^{s}, {}_{x}^{1}lfpr_{i(+)}^{s}, {}_{0}^{\infty}empr_{i(+)}^{+}, {}_{0}^{\infty}CJOBS_{i(+)}^{+}\right) \tag{9.58} $$

As discussed above, an additional labor-force adjustment is required to account for retirement patterns.

$$ {}_{65}^{\infty}postalfa_{i(+)}^{+} = \left({}_{x}^{1}PAGE_{i(+)}^{s}, {}_{x}^{1}P_{i(+)}^{s}, {}_{x}^{1}lfpr_{i(+)}^{s},\right) \tag{9.59} $$

After all the estimates years have been completed, the mean aged labor force adjustment should be calculated for each county, for eventual use as in an input the projections period. Let t_0 be the first estimate year, and t_e the last.

$$ {}_{0}^{\infty}alfa_{i(+)}^{+} = \sum_{t=t_0}^{t_e} \frac{{}_{65}^{\infty}postalfa_{i(+)}^{+}(t)}{t_e - t_0 + 1} \tag{9.60} $$

9.6.2 County-level Net Migration Projections

First, the working age population must be calculated.

$$ {}_{x}^{1}PWAO_{i(+)}^{s} = {}_{x}^{1}PAGE_{i(+)}^{s}, \text{ where } 16 \leq x \leq 80 \tag{9.61} $$

The labor force should then be determined. However, the adjustment for retirement-aged population must first be made. The adjusted labor force participation rates are denoted $alfpr$.

$$ {}_{x}^{1}alfpr_{i(+)}^{s} = \begin{cases} {}_{x}^{1}lfpr_{i(+)}^{s} \text{ if } x < 65 \\ {}_{0}^{\infty}alfa_{i(+)}^{+} * {}_{x}^{1}lfpr_{i(+)}^{s} \text{ if } x \geq 65 \end{cases} \tag{9.62} $$

Now, calculate the labor force.

$$ {}_{x}^{1}LF_{i(+)}^{s} = {}_{x}^{1}alfpr_{i(+)}^{s} * {}_{x}^{1}PWA_{i(+)}^{s} \tag{9.63} $$

Apply employment rates to yield the employed labor force.

$$ {}_{x}^{1}ELF_{i(+)}^{s} = {}_{0}^{\infty}empr_{i(+)}^{+} * {}_{x}^{1}LF_{i(+)}^{s} \tag{9.64} $$

Calculate maintenance employment.

$$ {}_{0}^{\infty}JMAINT_{i(+)}^{+} = {}_{0}^{\infty}jpe_{i(+)}^{+} * {}_{0}^{\infty}ELF_{i(+)}^{s} \tag{9.65} $$

Next comes the employment available for migrants.

$$ {}_{0}^{\infty}JMIG_{i(+)}^{+} = {}_{0}^{\infty}CJOBS_{i(+)}^{+} - {}_{0}^{\infty}JMAINT_{i(+)}^{+} \tag{9.66} $$

Determine the laboring labor migrants.

$$ {}_{0}^{\infty}LLTM_{i(1)}^{+} = \frac{{}_{0}^{\infty}JMIG_{i(+)}^{+}}{{}_{0}^{\infty}jpe_{i(+)}^{+}} \tag{9.67} $$

The laboring labor migrants must be scaled by the childhood dependency ratio to determine labor-tied migrants.

$$ {}_{0}^{\infty}LTM_{i(1)}^{+} = \left(1 + {}_{0}^{\infty}cdr_{i(1)}^{+}\right) * {}_{0}^{\infty}LLTM_{i(1)}^{+} \tag{9.68} $$

The retirement migrants for the state must then be apportioned to the counties. However, some counties have consistent negative net retirement migration, and simply distributing net migration proportionately will not maintain that pattern. Therefore, it becomes necessary to convert the net migration into a pseudo-gross migration. This is not a true gross migration, in terms of being only out-migration rates from counties to other counties. Instead, total state net retirement migration is converted to a piece that is for net in-counties, RMI and a piece that is for net out-counties, RMO. The net in total is apportioned to counties that are considered net in-counties based upon prior information. The net out total is similarly apportioned among the net out-counties. Additionally, for some counties there are insufficient data to determine whether it should be a net in or out county. For those cases, it is simplest to assume no net retirement migration.

So, we wish to calculate $RMI > 0$ and $RMO > 0$, as a function of RM, satisfying the identity $RM = RMI - RMO$. An infinite number of solutions are possible unless an additional identifying constraint is introduced.

We can assume that when in-retirement-migration to attractive areas increases, out-retirement-migration from less attractive areas will also increase. This might be related to a more fluid housing market, or to favorable timing to being drawing on retirement accounts. This means we wish to maintain some relationship where the in and out counties both increase or decrease together. This can be accomplished by a maintaining a ratio: the in-to-out ratio, or *ior*, defined as:

$$ {}_{65}^{\infty}ior_{+(1)}^{+} = \frac{{}_{65}^{\infty}RMI_{+(1)}^{+}}{{}_{65}^{\infty}RMO_{+(1)}^{+}} \tag{9.69} $$

which indicates cross-county "retirement mobility." The *ior* could be projected beforehand, but data constraints recommended holding *ior* constant based upon data observed between decennial censuses.

The following equations obtain ${}_{65}^{\infty}RMI_{+(1)}^{+}$ and ${}_{65}^{\infty}RMO_{+(1)}^{+}$ from ${}_{65}^{\infty}RM_{+(1)}^{+}$ and ${}_{65}^{\infty}ior_{+(1)}^{+}$.

First, the out:

$$ {}_{65}^{\infty}RMO_{+(1)}^{+} = \frac{{}_{65}^{\infty}RM_{+(1)}^{+}}{{}_{65}^{\infty}ior_{+(1)}^{+} - 1} \tag{9.70} $$

Then, the "in" can be calculated with either

$$ {}_{65}^{\infty}RMI_{+(1)}^{+} = {}_{65}^{\infty}RMO_{+(1)}^{+} + {}_{65}^{\infty}RM_{+(1)}^{+} \tag{9.71} $$

or, alternatively,

$$ {}_{65}^{\infty}RMI_{+(1)}^{+} = \frac{{}_{65}^{\infty}ior_{+(1)}^{+} * {}_{65}^{\infty}RM_{+(1)}^{+}}{{}_{65}^{\infty}ior_{+(1)}^{+} - 1} \tag{9.72} $$

Each county must be identified as either an "in" or "out" or "none" retirement county. This is stored in a variable called *rtype*, which equals 1 for "in" counties, 2 for "out" counties, and 3 for "none" counties. The "in" counties must have a proportionate frequency distribution, *rpi*, that distributes *RMI*. Similarly, the "out" counties must have a frequency distribution *rpo* for distributing *RMO*. The categorization for each county should be determined beforehand, as should the *rpi* and *rpo* distributions. Note the net migration for the "out" counties must be made negative.

The total net retirement estimate for each county is,

$$ {}_{65}^{\infty}RM_{i(1)}^{+} = \begin{cases} -{}_{0}^{\infty}rpo_{i(1)}^{+} * {}_{0}^{\infty}RMO_{+(1)}^{+} & \text{if } rtype = 1 \\ {}_{0}^{\infty}rpi_{i(1)}^{+} * {}_{0}^{\infty}RMI_{+(1)}^{+} & \text{if } rtype = 2 \\ 0 & \text{if } rtype = 3 \end{cases} \tag{9.73} $$

The final total general population age 65+ can now be calculated.

$$ {}_{65}^{\infty}PO65_{i(1)}^{+} = {}_{65}^{\infty}RM_{i(1)}^{+} + {}_{65}^{\infty}PAGE_{i(1)}^{+} \tag{9.74} $$

From this point onward, the process is identical to that used during the estimates period. The quantities to be calculated are as detailed in equations 9.48 - 9.58.

Chapter 10
Households

Households are important for many aspects of policy planning, because a household is often considered the unit of economic consumption. With respect to these consumption units, the resident population can be divided into two groups - households and group quarters. The Census Bureau measures several types of group quarters. In Utah, examples include prisons, military barracks, and the Missionary Training Center. Households (and group quarters) are projected at the county level, and are completed after the population projections.

First, group quarters are projected. UDEM takes inputs of the group quarters population by age and sex at the time of the decennial census for each county. These must be determined beforehand, and categorized into one of three types. The projection method then differs based upon the type of group quarters.

The household population is the population that does not live in group quarters. The total number of households is calculated by applying age-specific rates of being a head-of-household to the household population, and then finding the total. Household size is simply the average number of people in a household. The county numbers are used to find the state totals.

10.1 Group Quarters

Groups quarters can be divided into three stock groups.

1. Constant group quarters: are unchanging stocks each year, and represented by ${}_{x}^{1}CGQ_{i(+)}^{s}$.
2. Share group quarters: considered to maintain a constant share of the population, and is represented by ${}_{x}^{1}SGQ_{i(+)}^{s}$.
3. "Middle" group quarters: considered to be "halfway" between an unchanging enumerations and unchanging shares, represented by ${}_{x}^{1}MGQ_{i(+)}^{s}$.

These three options permit a reasonable degree of flexibility for incorporating expert knowledge about changes in Utah's different group quarters populations, while maintaining a reasonable degree of parsimony.

We assume that the initial values are determined from the decennial census. Let the decennial population be denoted ${}_{x}^{1}DECP_{i}^{s}$. We must first calculate shares at the census for the non-constant group quarters:

$$ {}_{x}^{1}sgqr_{i(+)}^{s} = \frac{{}_{x}^{1}SGQ_{i(+)}^{s}(t=0)}{{}_{x}^{1}DECP_{i(+)}^{s}} \tag{10.1} $$

$$ {}_{x}^{1}mgqr_{i(+)}^{s} = \frac{{}_{x}^{1}MGQ_{i(+)}^{s}(t=0)}{{}_{x}^{1}DECP_{i(+)}^{s}} \tag{10.2} $$

Recall from the notation chapter that t=0 at the census. The group quarters can then be calculated at each successive time point as:

$$ {}_{x}^{1}SGQ_{i(+)}^{s} = {}_{x}^{1}sgqr_{i(+)}^{s} * {}_{x}^{1}P_{i(+)}^{s} \tag{10.3} $$

$$ {}_{x}^{1}CGQ_{i(+)}^{s} = {}_{x}^{1}CGQ_{i(+)}^{s}(t=0) \tag{10.4} $$

$$ {}_{x}^{1}MGQ_{i(+)}^{s} = \frac{{}_{x}^{1}MGQ_{i(+)}^{s}(t=0) + {}_{x}^{1}mgqr_{i(+)}^{s} * {}_{x}^{1}P_{i(+)}^{s}}{2} \tag{10.5} $$

The sum of the the three group quarters types yields the total,

$$ {}_{x}^{1}GQ_{i(+)}^{s} = {}_{x}^{1}CGQ_{i(+)}^{s} + {}_{x}^{1}MGQ_{i(+)}^{s} + {}_{x}^{1}SGQ_{i(+)}^{s} \tag{10.6} $$

10.2 Households

The household population is calculated as the residual of the group quarters with total population.

$$ {}_{x}^{1}HHP_{i(+)}^{s} = {}_{x}^{1}P_{i(+)}^{s} - {}_{x}^{1}GQ_{i(+)}^{s} \tag{10.7} $$

Age-specific headship rates are defined as the proportion of the household population aged x to $x+h$ that is the head of a household. UDEM requires single-year-of-age-specific headship rates, denoted ${}_{x}^{1}headr_{i(+)}^{+}$ to be projected beforehand, and input exogenously.

The headship rates are multiplied by the household population to yield households by age of the head (agnostic to sex):

$$ {}_{x}^{1}HH_{i(+)}^{+} = {}_{x}^{1}headr_{i(+)}^{+} * {}_{x}^{1}HHP_{i(+)}^{+} \tag{10.8} $$

The total households are simply summed over age x. The state values are obtained by summing over index i.

10.3 Persons per Household

Persons per household is obtained by dividing the total household population by the total number of households.

$$ {}_{0}^{\infty}pph_{+(+)}^{+} = \frac{{}_{0}^{\infty}HHP_{+(+)}^{+}}{{}_{0}^{\infty}HH_{+(+)}^{+}} \tag{10.9} $$

Note that the state-level persons per household must be calculated in this manner—it cannot simply be summed from the county persons per household, because forces don't aggregate.

Chapter 11
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